Subject: Mathematics
Developer Name: Warren Lofgren/Joe Carey
Unit Title: Rate of Change
Location:

Course Description:
This unit is intended for students in courses preceding Calculus. Activities 1 and 2 could be used in an Algebra I course. The entire unit could be used as part of an Algebra II course or the course immediately preceding Calculus.

Unit Context:
Students need to be very comfortable with function notation, plotting points on a coordinate plane, using a graphing calculator to analyze a function, and calculating slope.

Length of Unit:
Assuming a 45-55 minute daily class period, this unit has five class periods of lessons. If the class spends a lot of time discussing the student assignments, as many as seven days may be required to complete this unit. The first two lessons could be used as two one day lessons in a first year high school math class such as Algebra I to introduce the concept that slope describes the rate of change in a linear situation.

Iowa Tests Core Content Standards and Benchmarks

Core Content Standards and Benchmarks

• Standard A: Students can understand and apply a variety of math concepts.
  o Benchmark 1: Students can understand and apply number properties and operations.
  o Benchmark 2: Students can understand and apply concepts and procedures of algebra.
• Standard C: Students can solve a variety of math problems.
  o Benchmark 1: Students can solve math problems requiring multiple steps and operations.
  o Benchmark 2: Students can reason quantitatively.
• Standard D: Students can interpret data presented in a variety of ways.
  o Benchmark 1: Students can make inferences based on data presented in a variety of ways.
  o Benchmark 2: Students can interpret data from a variety of sources.

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ICC Essential Concepts, Skills, and Characteristics

Concepts

Essential Concepts from the Iowa Core Curriculum

- .....in Algebra:

  - ☑ Understands and applies functions
  - ☑ Understands and applies equations and inequalities
  - ☑ Understands and applies algebraic expressions
  - ☑ Understands and applies rate of change
  - ☑ Understands and applies recursion and iteration

- ..... in Geometry

  - ☑ Understands and applies coordinates

- ....in Quantitative Literacy
  - ☑ Understands and applies number operations and properties.

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Skills

Essential Skills from the Iowa Core Curriculum

- .....in Problem Solving:

  - ☑ Builds new mathematical knowledge through problem solving
  - ☑ Applies and adapts a variety of appropriate strategies to solve problems in mathematics and other contexts
  - ☑ Monitors and reflects on the process of mathematical problem solving

- .....in Communication:

  - ☑ Organizes and consolidates his/her mathematical thinking through communication
  - ☑ Communicates his/her mathematical thinking coherently and clearly to peers, teachers, and others
  - ☑ Analyzes and evaluates the mathematical thinking and strategies of others
  - ☑ Uses the language of mathematics to express mathematical ideas precisely

- .....in Reasoning and Proof:

  - ☑ Makes and investigates mathematical conjectures

- .....in Representation:

  - ☑ Creates and uses representations to organize, record, and communicate mathematical

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ideas
- ☑ Selects, applies, and translates among mathematical representations to solve problems
- ☑ Uses representations to model and interpret physical, social, and mathematical phenomena

- ……in Connections:
  - ☑ Recognizes and applies mathematics in contexts outside of mathematics

Characteristics

- Encourages collaboration in learning
- Teaches for understanding
- Student centered
- Develops conceptual and/or procedural knowledge
- Provides authentic learning
- Teaches through problem solving & inquiry
- Assesses for learning
- Incorporates current technology
- Provides global perspective

Big Ideas Addressed in Each Activity

Activity 1: The Hot Tub

- Recognize the intervals on which a graph is increasing and decreasing
- Recognize the rates of increase and decrease of a graph
- Provide real world interpretation of changes in a graph

Activity 2: Walk This Way

- Interpret slope(s) of a time vs. distance graph
- Analyze a time vs. distance graph in context
- Generalize slope to rate of change in other contexts

Activity 3: How Many Handshakes?

- Creates a recursive model of a real life phenomena using NOW/NEXT and H(n)/H(n-1) notation
- Recognizes that situations in which the second difference is a constant can be modeled with quadratic functions
- Given 3 ordered pairs, finds the quadratic function passing through these points

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Activity 4: Cliff Diver

- Average rates of change for non-linear (quadratic) functions
- Estimates rates of change at a particular moment
- Determines intervals on which a function is increasing or decreasing
- Gives real world interpretation to rates of change in a graph

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Activity 5: Fish Population
◦ Estimates rates of change at a particular moment ◦ Calculates the instantaneous rate of change using the derivative formula and the graphing calculator ◦ Gives real world interpretation to rates of change in a graph ◦ Interprets shape and meaning of the logistic curve

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Rigor & Relevance Plot: Rate of Change

RIGOR

Evaluation

Synthesis

Analysis

Application

Comprehension

Knowledge

Activity 3: The Handshake Activity Students build recursive and explicit formulas to determine the number of handshakes required for n people in a room to shake hands with every other person once. **Activity 4:** Cliff Diver Students use slopes of secant lines to approximate slopes of tangent lines. **Activity 5:** Fish Population Students analyze a function in context to interpret its meaning and application. QUADRANT C	**Activity 1:** The Hot Tub. Students create their own graph and accompanying story of a real world situation. **Activity 4:** Cliff Diver Student approximate the rate a cliff diver hits the water. **Activity 5:** Fish Population Students approximate the population rate of change at different times, by analyzing the graph, table, and function rules. QUADRANT D
Activity 1: The Hot Tub Students use a graph to tell a story about what may be happening in some real world situation. **Activity 2:** Walk This Way Students compute slope of a position versus time graph. QUADRANT A	**Activity 1:** The Hot Tub Students use a graph to tell a story about what may be happening in some real world situation. **Activity 2:** Walk This Way Students use slope of a position versus time graph to describe speed of a walker. **Activity 5:** Fish Population Students analyze a graph to interpret shape in context. QUADRANT B

Knowledge in one discipline | Application in one discipline | Interdisciplinary application | Real-world predictable application | Real-world unpredictable application

RELEVANCE

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Concept Map:

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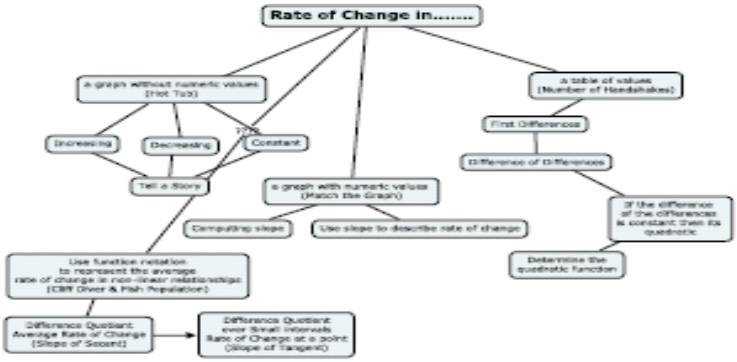

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ACTIVITY 1

Activity 1 Overview: Hot Tub

Iowa Core Curriculum Essential Skills/Concepts:	Essential Content from the Iowa Core Curriculum • … in Algebra: ☑ Understands and applies functions ☑ Understands and applies rate of change • …in Geometry ☑ Understands and applies coordinates Essential Skills from the Iowa Core Curriculum • …in Problem Solving: ☑ Builds new mathematical knowledge through problem solving ☑ Applies and adapts a variety of appropriate strategies to solve problems in mathematics and other contexts • …in Communication: ☑ Organizes and consolidates his/her mathematical thinking through communication ☑ Communicates his/her mathematical thinking coherently and clearly to peers, teachers, and others ☑ Analyzes and evaluates the mathematical thinking and strategies of others • …in Representation: ☑ Creates and uses representations to organize, record, and communicate mathematical ideas ☑ Uses representations to model and interpret physical, social, and mathematical phenomena • … in Connections: ☑ Recognizes and applies mathematics in contexts outside of mathematics
Big Ideas	° Recognizes the intervals on which a graph is increasing and decreasing ° Recognizes the rates of increase and decrease of a graph ° Gives real world interpretation of changes in a graph

Characteristics of Instructional Core		Important core content that is evidence-based		Deep conceptual and procedural knowledge	X	Reflective teaching in a collaborative setting
	X	Teaching for understanding		Rigorous curriculum		Development of global perspective
	X	Teaching through problem solving and inquiry	x	Relevant curriculum		Incorporation of current technology
	X	Student-centered classrooms	x	Assessment for learning		
Cognitive Domain		Remembering	x	Applying		Evaluating

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		Understanding	x	Analyzing	x	Creating
Connections to Students' Lives		Isolated within discipline		Connected to other disciplines	x	Connected to student lives
	X	Connected within discipline	x	Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		
		Increasing access to print		Engaging students with texts		
	X	Involving students in discussion		Reading aloud in content areas		
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension		
	X	Writing to learn across content areas				
Class Time	1 class period	45-55 total minutes				

Materials & Set-up:
A worksheet and assignment sheet for each student is needed. An electronic copy of the worksheet in Geometer's Sketchpad (GSP) to project on a whiteboard would be nice. If this isn't available, the graph on the student handout can be placed on an overhead and projected for all to see. Be prepared to describe a situation that could be occurring in the hot tub that is modeled by the given graph.

Teaching Tips:
Dynamic software graphing software such as Geometer's Sketchpad makes it much easier to ask "what if" questions.

Teacher Tasks	Student Tasks
• Create the "Hot Tub Graph" in GSP or find another way to display the "Hot Tub Graph."	• Participate in discussion about the "Hot Tub" story.
• Lead a discussion around analyzing the graph in order to tell an initial story about the hot tub.	• Create alternative "Hot Tub" story.
• Throughout the discussion encourage alternative explanations.	• Copy partner's graph.
• Ask "What if…" questions and show the result by moving points to alter the rate of change for various segments and investigate how changing the story would change the graph . (E. g., what if a second hose was added during the fill time? What if three people got in the tub instead of one?)	• Complete assignment related to graph analysis.
• Create a situation by moving points on the graph that illustrate an impossible physical situation (a graph that fails the vertical line test.) .	• Participate in discussion about the "Hot Tub" story.
• After the students have answered question 9, have them share their story with another student.	• Create alternative "Hot Tub" story.

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Differentiation Tasks
Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction: • vocabulary: rate of change, slope • skills/concepts: calculating slope, positive/negative/zero Supplemental instruction suggestions: • Strategic heterogeneous partnerships/learning groups • The concept of volume and displacement of water is probably understood by all students, but it may be possible that a few students could need manipulatives to model the situation (beaker and marbles?) • For the student work assignment, Question 1 should read "Create a graph that includes line segments that are increasing, decreasing, and staying constant. Write a story that the graph illustrates. The story can be about any situation other than filling hot tubs." (Number of segments not as critical as demonstrating understanding of increasing, decreasing, constant). Question 2 should scaffold the calculation of slope ("recall that slope is calculated by finding the change in the y axis variable (rise) and dividing by the change in the x axis variable (run)). Less-than-proficient students could be asked to create and analyze graphs with fewer line segments. • Highly proficient students could be asked to put reasonable numeric values on the vertical and horizontal axes and compute the rate of change for various line segments.

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Assessment:

Formative Assessments:

- A formative assessment of the students' understanding can be made as the discussion of the original graph occurs class.

Summative Assessments:

- The students should create a new graph that describes a different hot tub situation and create a story to go with their graph. This would provide a summative assessment of the students' learning.

Reflect:

Comment [MB2]: Needs to be completed

Many students enjoyed the opportunity to be creative in a math class. Their stories were varied and often entertaining. Reminding students to keep the stories "school appropriate" is an important first step for this activity. Typical stories for the hot tub situation for the first graph went something like this. At point A the hot tub is empty, but the water is turned on. The rate at which the tub is being filled is constant from A to B. Between B and C the water enters the tub at a faster rate. From C to D there is no change in the depth of the water in the tub indicating the water was turned off. From D to E water is drained from the tub. At point F someone entered the hot tub causing the water level to rise very quickly. We spent a great deal of time discussing the fact that this line cannot be vertical. It only appears to be vertical because the time required to fill a hot tub is measured in a fraction of an hour and the time required to enter a hot is a fraction of a second. From point G to point H the person in the hot tub relaxes so there is no change in water level. From point H to point I the person who entered the hot tub stands up causing the water level to fall. Between points J and K the person steps completely out of the hot tub. Between points K and L the hot tub is drained.

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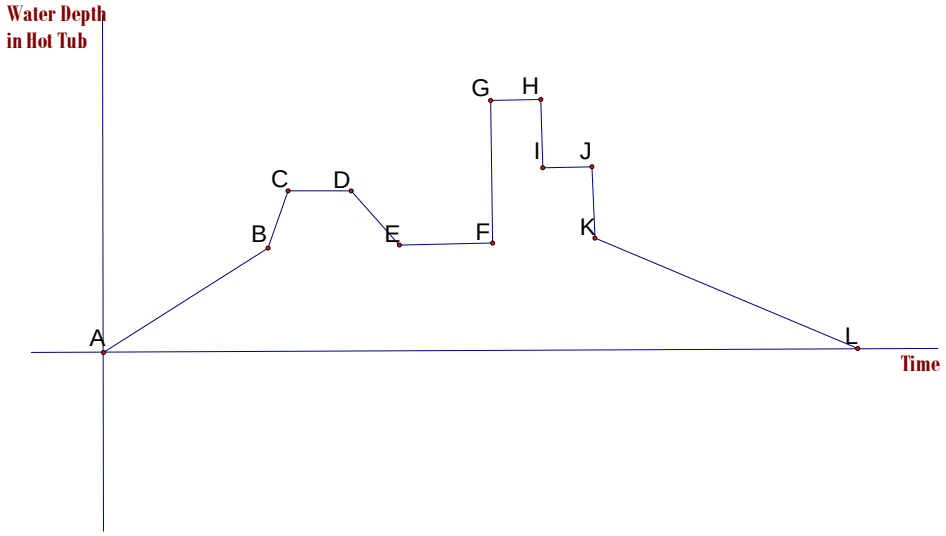
Attachments:

Student Page/Handout

Comment [MB3]: Are vertical lines physically possible?



1. If the tub is being filled from point A to point B, which other intervals represent a time when the water level in the tub is increasing?
2. On which intervals is the water level in the tub decreasing?
3. When is the water level increasing the fastest?
4. When is the water level decreasing the fastest?
5. On which intervals does the water level remain constant?
6. Create a story to go with your description of for each of the line segments.
7. Can the water level be two different heights at the exact same time?
8. At what point(s) on the graph does it appear that the graph shows the hot tub having two different water levels at the same moment? Explain why this might appear to be happening.

Comment [MB4]: I can't answer this question.

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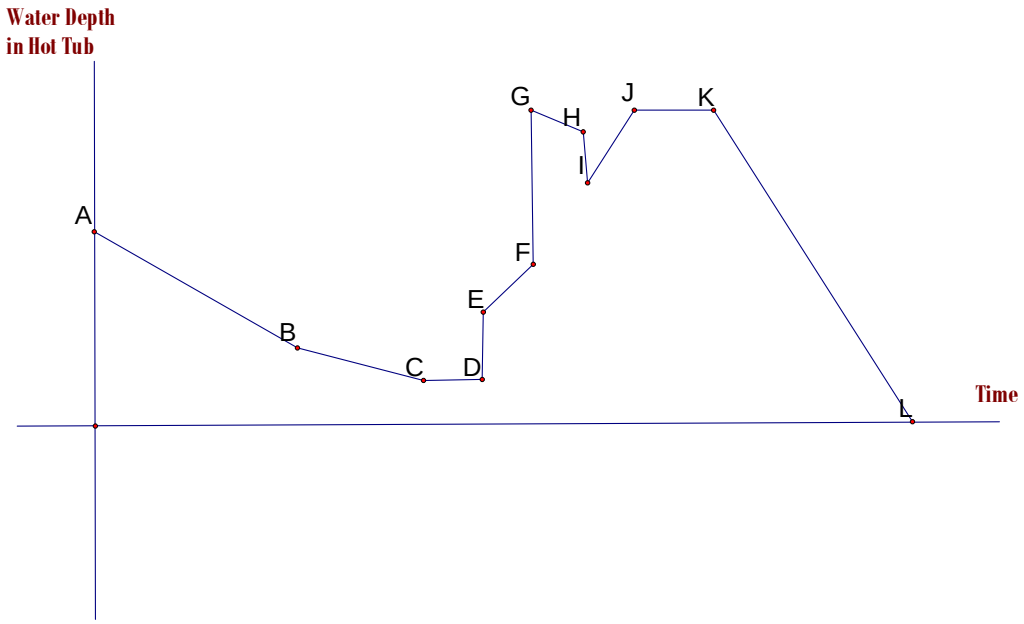
9. If the vertical axis represents the amount of money in a bank account, create a story that could be described by this graph.

Comment [MB5]: If, instead, vertical



10. If the tub is being drained from point A to point B, which other intervals represent a time when the water level in the tub is increasing?
11. On which intervals is the water level in the tub decreasing?
12. When is the water level increasing the fastest?
13. When is the water level decreasing the fastest?
14. On which intervals does the water level remain constant?
15. Create a story to go with your description of for each of the line segments. Share this story with a classmate. Have them critique your story while you critique his/her story.
16. Can the water level be two different heights at the exact same time?
17. At what point(s) on the graph does it appear that the graph shows the hot tub having two different water levels at the same moment? Explain why this might appear to be happening.
18. If the vertical axis represents the amount of money in a bank account, create a story that could be described by this graph. Share this story with a classmate. Have him/her critique how well your story and your graph match while you critique his/her story.

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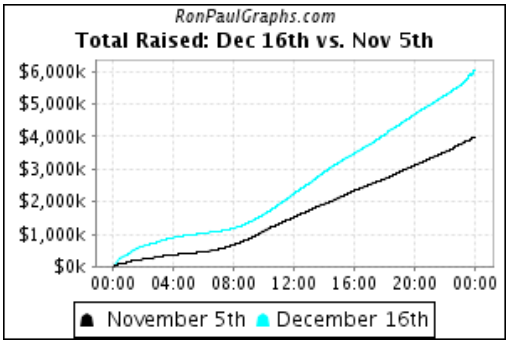
Student work

The assignment for the students is made up of two tasks.

1. Create a graph which has at least seven line segments and a story to go with the graph. The graph should have segments that are increasing, decreasing, and constant. The story can deal with any situation other than filling hot tubs and putting money into a bank account.

2. The graph shown below describes the amount of money raised by 2008 presidential candidate Ron Paul during two days in late 2007. Determine the day and time frame when the contributions were coming in the fastest. What was the rate at which the contributions were coming in at this moment?



Comment [MB6]: I like this activity, except for the vertical lines. I would love calc students who came into my class having previously done this activity.

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ACTIVITY 2

Activity 2 Overview: Walk This Way

Iowa Core Curriculum Essential Skills/Concepts:	° Builds new mathematical knowledge through problem solving ° Communicates mathematical thinking coherently and clearly to peers, teachers, and others ° Makes and investigates mathematical conjectures ° Recognizes and applies mathematics in contexts outside mathematics ° Uses representations to model and interpret physical, social, and mathematical phenomena					
Big Ideas	° Interpret slope(s) of a time vs. distance graph. ° Analyze a time vs. distance graph in context. ° Generalize slope to rate of change in other contexts.					
Characteristics of Instructional Core	X	Encourages collaboration in learning	X	Teaches for understanding		Develops global perspectives
	X	Student centered	X	Develops conceptual and/or procedural knowledge	X	Provides authentic learning
	X	Teaches through problem solving & inquiry		Assesses for learning	X	Incorporates current technology
Cognitive Domain		Remembering		Applying		Evaluating
		Understanding		Analyzing		Creating
Connections to Students' Lives		Isolated within discipline	X	Connected to other disciplines		Connected to student lives
	X	Connected within discipline	X	Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		
		Increasing access to print		Engaging students with texts		
		Involving students in discussion		Reading aloud in content areas		
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension		
		Writing to learn across content areas				
Class Time	0 class period(s) 0 total minutes					

Materials & Set-up:

A set of pre-made time vs. distance graphs, either electronic form or on transparency
A TI-83/84 graphing calculator connected to a motion detector such as a CBR.
A method of displaying the calculator screen to the class (TI-Presenter or other device)
A computer with a projector to display Geometer's Sketchpad file
The website www.explorelearning.com has a "gizmo" that illustrates these ideas well

Comment [MB7]: TI may be phasing these out. You should check.

Teaching Tips:

This activity can be launched by discussing the idea of a "radar gun" used by police to detect speeding motorists. The discussion can lead into the activity of using the motion detector in class to create our own

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time vs. distance graphs to be analyzed and interpreted. To begin, have students examine a pre-made graph and think of how a person would walk to recreate that graph. Test their conjecture by using the motion detector. It may take several attempts, but will provide for good discussion as to what the student is doing or not to cause them to not match the given graph. Use the other pre-made graphs to ask the same thing, and have volunteer students try to walk in front of the motion detector to replicate the given graph. Make the connection between the slopes of the line segments with the rate of change (feet/sec) of the walker. With the Sketchpad file, students can easily create a time vs. distance graph to be interpreted and "walked" by another classmate, or the graph can be manipulated by the teacher to create different situations.

Teacher Tasks	Student Tasks
• Provide motivating questions about how radar detectors work to lead into the motion detector activity • Set up the motion detector and demonstrate its use to class • Display an initial graph for students to discuss how to walk to recreate the graph • Play "Walk This Way" by Aerosmith and have a student volunteer try to match the given graph • Point out differences between the walked graph and the given graph and lead discussion in how to better match the two • Ask students how the visual aspect of the graph corresponds to the physical motion of walking, leading to the connection between slope and rate of change (feet/sec) • Discuss methods, and symbolism of calculating and interpreting slope: rise/run, $\frac{y_2 - y_1}{x_2 - x_1}$, $\frac{\Delta y}{\Delta x}$, speed and direction of walker • Demonstrate to class how to operate the Distance-Rate-Time "gizmo" on the website www.explorelearning.com	• Participate in discussion about radar detectors, volunteering their knowledge and experiences with one • Examine graphs of distance vs. time for aspects such as speed, changes in speed, starting distances, etc. • Walk in front of the detector to try to match a given distance vs. time graph • Observe classmates attempting to replicate a graph to see what, if any, changes should be made to their speed and direction • Create new graphs on the Sketchpad file for other students to attempt to replicate • Answer questions about interpreting a distance vs. time graph regarding the speed, direction, and starting distance of a walker
Differentiation Tasks	
Have some pre-made graphs that involve fewer pieces to replicate. Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction:	

Comment [MB8]: Will this offend right-wing people? ☺

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- slope, rate of change, increasing, decreasing

Supplemental instruction suggestions:

- Students who are less-than-proficient with rate of change usually respond well to the CBRs because it makes the concept very concrete. However, sometimes they learn this context too well and over generalize (e.g., a line that goes down indicates moving towards something and a line going up indicates moving away from something). Check in with students to make sure they avoid this pitfall by asking questions like "What if this was a graph of "total distance" instead of "distance from source"? Is it possible for this line to go down in that context?"
- Encourage students to use the gizmo from www.explorelearning.com to check and test out their conjectures about matching graphs to stories.

Highly Proficient

Supplemental instruction suggestions:

- Students who are highly proficient may not need the activity with the CBRs to understand rate of change, but usually find it motivating to experiment with the technology anyway. Their activity and assignment could be more focused on creating graphs that are physically impossible to replicate and creating non linear graphs.
    - Create a time vs. distance graph that would be impossible to physically replicate and explain why. (supplemental)
    - Create a time vs. distance graph that involves non-linear pieces and have students interpret. (supplemental)

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Comment [MB9]: I love this.

Assessment:

Formative Assessments:

- Observation of students attempting to replicate the given graph while walking in front of the detector will indicate whether a student understands the main ideas derived from the graph
- In-class discussion of the graphs will indicate whether students understand how to interpret the graph even if they are unable to "walk" the graph with the detector
- The activity can be reversed. Give students a written or verbal description of a walk, and ask them to graph it. For example, "A student begins 5 ft from the detector and walks away at a speed of 2 ft/sec for 4 seconds, then they stand still for 3 seconds, …"

Summative Assessments:

- Have students complete an assignment with distance vs. time graphs given, where questions are asked that require students to interpret the graph. Example questions are given below. Students also would be asked to take a written interpretation and create the graph.

Reflect:

Students enjoyed the challenge of physically replicating a given distance vs. time graph. They were engaged and even excited to try this. It provided an activity to refer back to. It also provided some humor.

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Attachments:

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Student Page/Handout (if appropriate)

Questions to ask when interpreting a Distance vs. Time graph

1. How far does the walker start from the detector? How do you know that from looking at the graph?

2. How fast is the walker walking in the first time interval (from point A to point B)? Be sure to show how you arrived at your answer and include units.

3. What is the slope of the line segment from A to B? How does this compare to the speed you found in question 2?

4. Are there any intervals where the walker is standing still? If so, when are they? If not, what would the graph have to look like to represent standing still?

5. During which time interval does the walker have the greatest speed? What is that speed?

6. Are there any intervals where the walker is walking towards the detector, rather than away from it? When are they?

7. How do intervals where the walker is walking towards the detector show up differently on the graph than those intervals where the walker is moving away from the detector?

8. How does a faster speed change the visual appearance of an interval on the graph?

Use the written interpretation below to sketch a graph of a walker on the grid given.

Starting from 6 feet away from the detector, Paul moves at a speed of 3 ft/sec for a total of 4 seconds. He then slows down to 1 ft/sec for 5 seconds. After this brutal pace, he is tired and stands still for 5 seconds to catch his breath. Turning around, he walks back towards the detector at a speed of 8 ft/sec for 2 seconds. Paul gets spooked by the detector and turns around and runs at a speed of 12 ft/sec for 2 more seconds.

Screenshot of the Geometer's Sketchpad file to accompany the Walk This Way activity.

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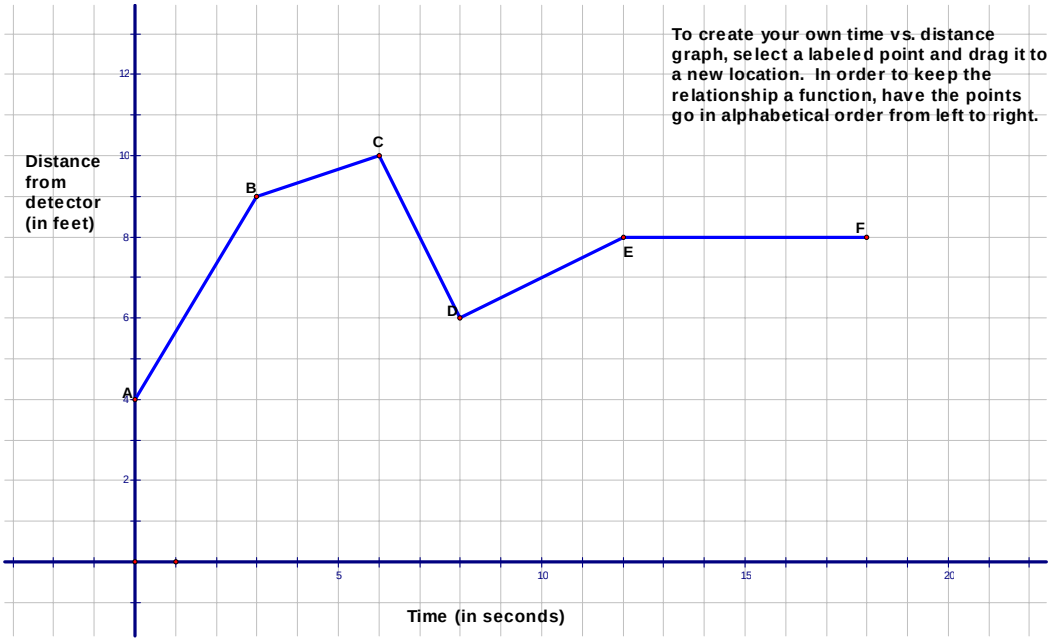
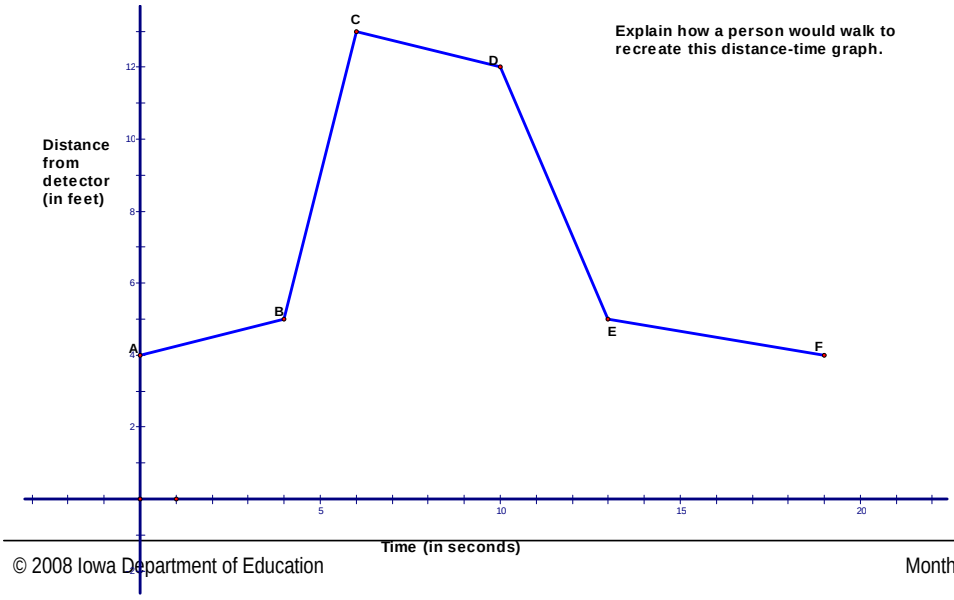


This is a picture of the Sketchpad file that allows students to manipulate the points to create their own time vs. distance graphs. Students could create graphs for others to try to replicate and the teacher could use this for demonstration and discussion purposes. It would easily allow the teacher to pose "what if…" questions to the class and to change the appearance of the graph to fit different situation and conditions to assess student understanding.

Time vs. Distance Graph 1



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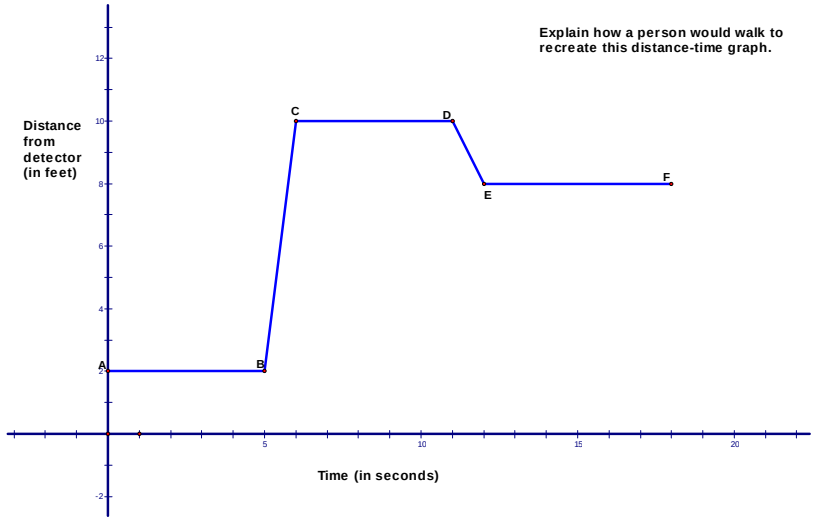
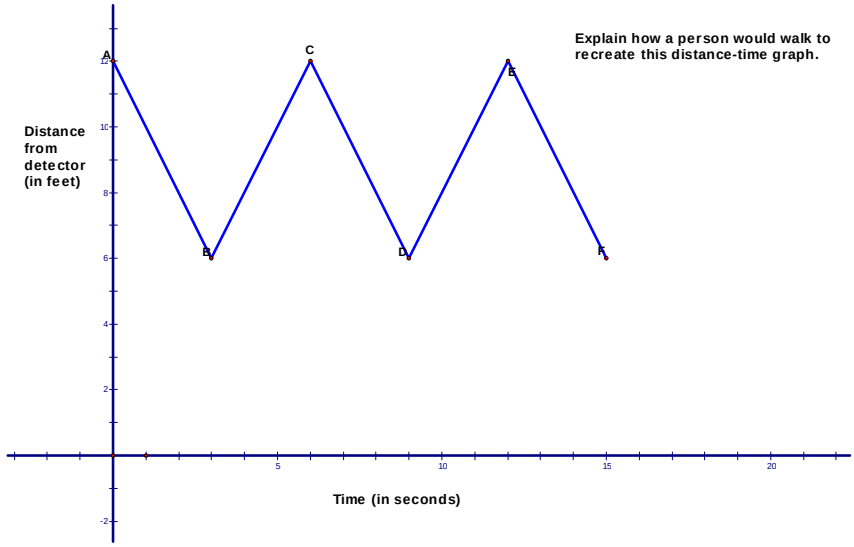
Time vs. Distance Graph 2



Time vs. Distance Graph 3



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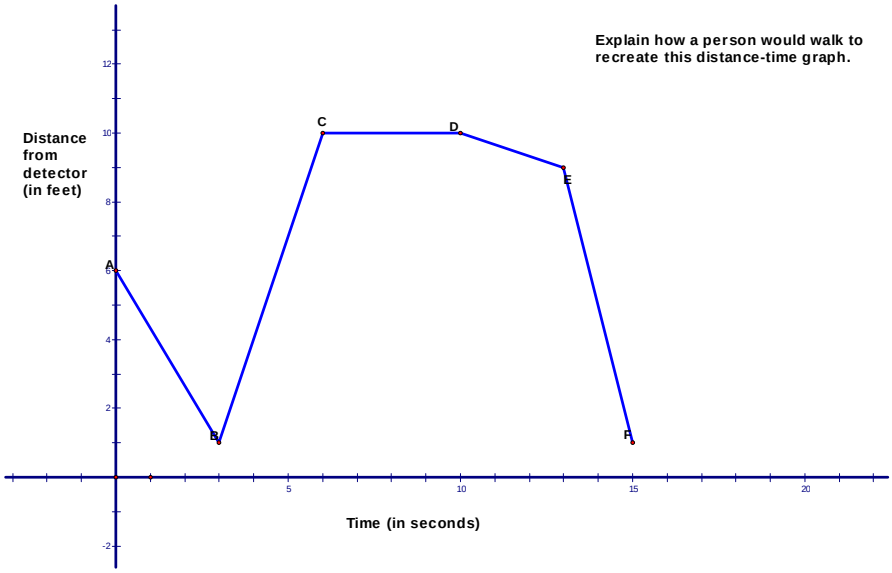
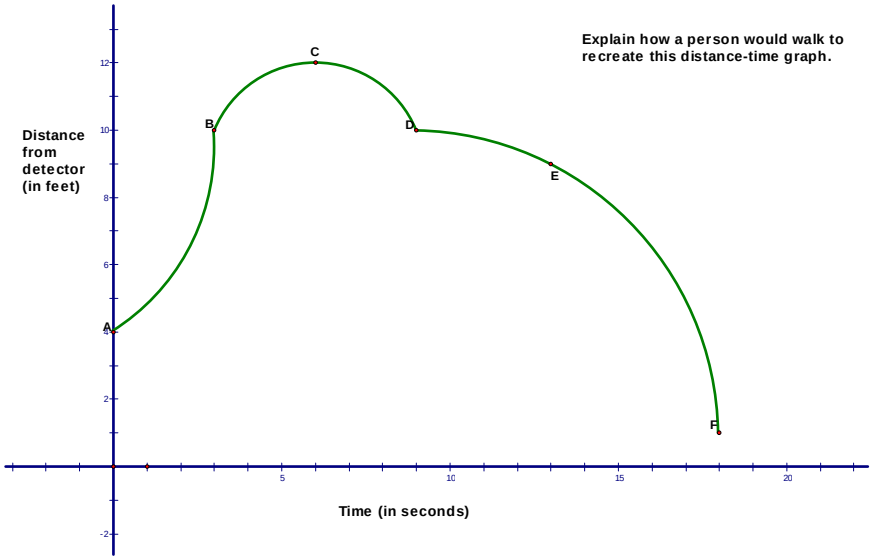
Time vs. Distance Graph 4



Time vs. Distance Graph 5



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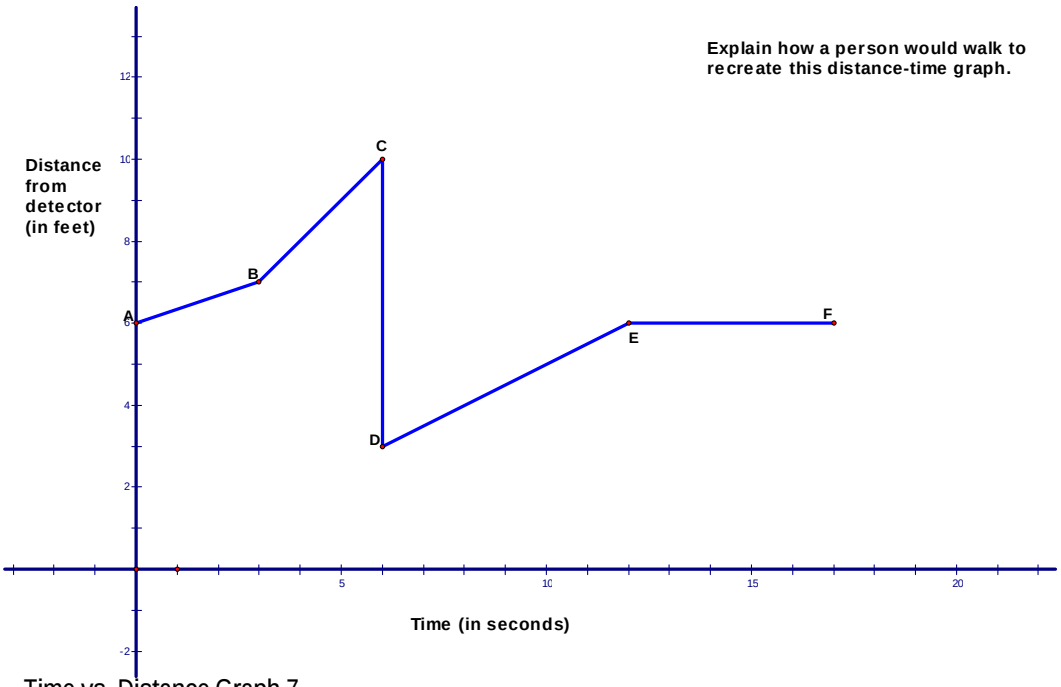
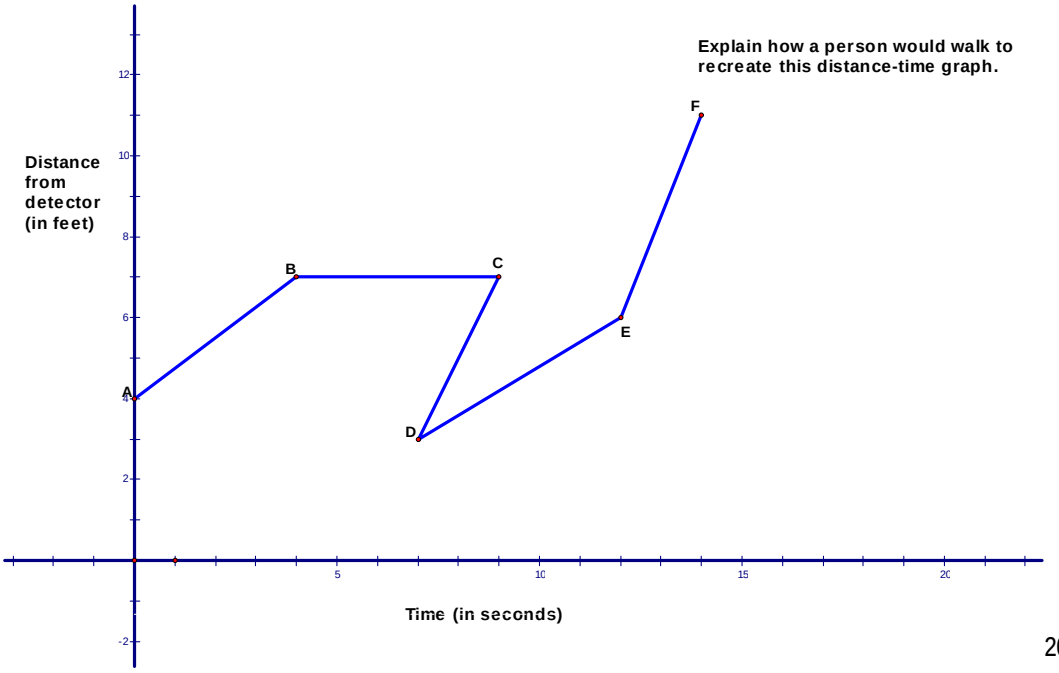
Time vs. Distance Graph 6



Time vs. Distance Graph 7



Comment [MB10]: Currently, most of the graphs have corners or "cusps". It might be nice to discuss what is happening at a cusp, and if it is possible to make one in the real world. (Good forshadowing of differentiability) Another great exercise!

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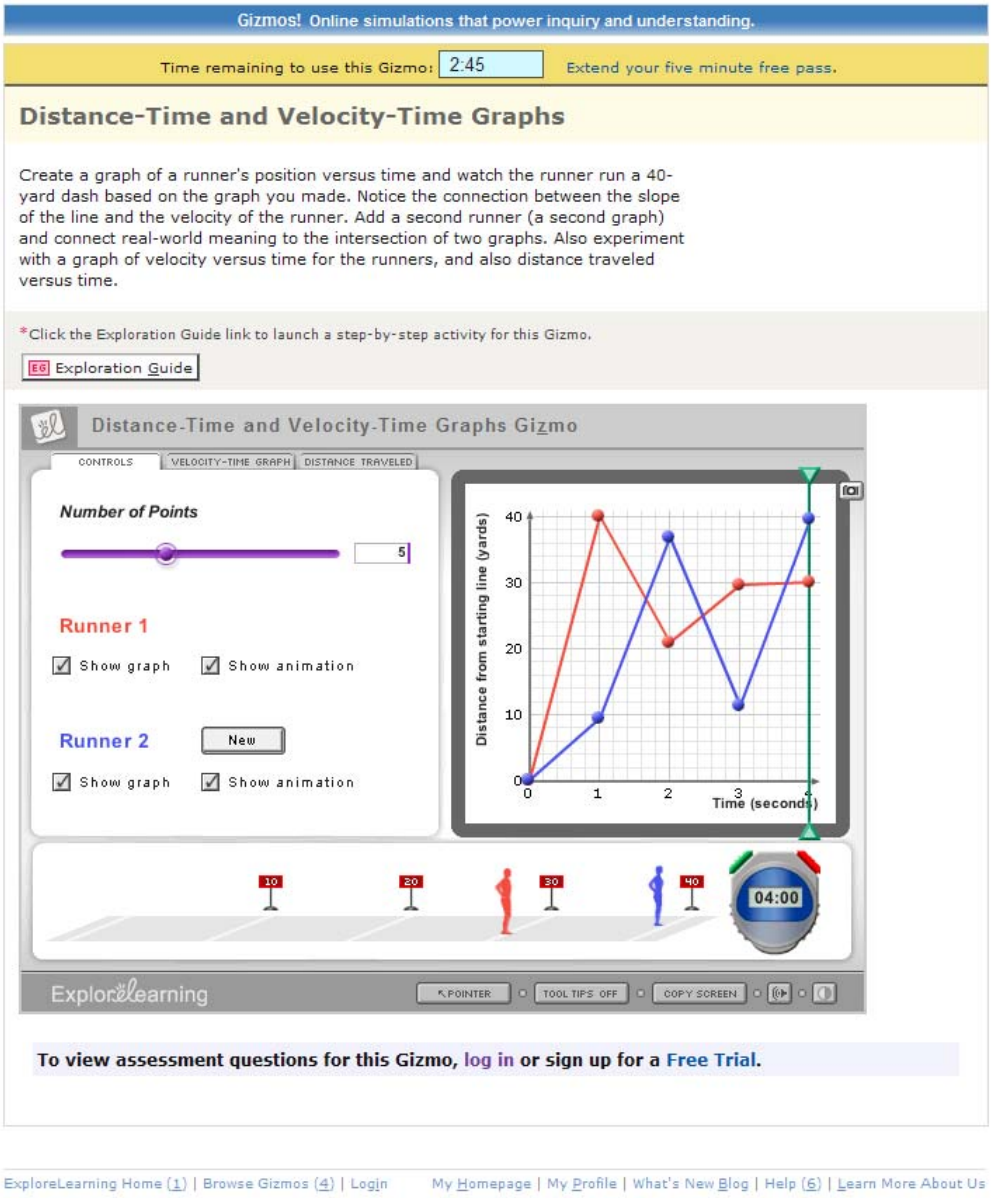
Screenshot of the "gizmo" on Distance-Time graphs from the website www.explorelearning.com

Comment [MB11]: Is it easy to find, or do we need more directions?



ExploreLearning Home (1) | Browse Gizmos (4) | Login My Homepage | My Profile | What's New Blog | Help (6) | Learn More About Us

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This website activity can be used as an extension to the in-class activity. It allows the class to create dynamic distance-time graphs, similar to the Sketchpad file. However, it goes one step further than the Sketchpad file as it allows you to actually have a simulated runner act out the distance-time graph so the student can see what the graph "looks" like as it is done. It also allows you to race two runners against each other which permits students to create different outcomes and scenarios by manipulating the graph.

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ACTIVITY 3

Activity 3 Overview: How Many Handshakes?

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Iowa Core Curriculum Essential Skills/Concepts:	Essential Skills from the Iowa Core Curriculum • ……in Problem Solving: ☑ Builds new mathematical knowledge through problem solving ☑ Applies and adapts a variety of appropriate strategies to solve problems in mathematics and other contexts ☑ Monitors and reflects on the process of mathematical problem solving • .....in Communication: ☑ Organizes and consolidates his/her mathematical thinking through communication ☑ Communicates his/her mathematical thinking coherently and clearly to peers, teachers, and others ☑ Analyzes and evaluates the mathematical thinking and strategies of others ☑ Uses the language of mathematics to express mathematical ideas precisely • ……in Reasoning and Proof: ☑ Makes and investigates mathematical conjectures • ……in Representation: ☑ Creates and uses representations to organize, record, and communicate mathematical ideas ☑ Selects, applies, and translates among mathematical representations to solve problems ☑ Uses representations to model and interpret physical, social, and mathematical phenomena • ……in Connections: ☑ Recognizes and applies mathematics in contexts outside of mathematics Essential Concepts from the Iowa Core Curriculum • .....in Algebra: ☑ Understands and applies functions ☑ Understands and applies equations and inequalities ☑ Understands and applies algebraic expressions ☑ Understands and applies rate of change ☑ Understands and applies recursion and iteration • ..... in Geometry ☑ Understands and applies coordinates • .....in Quantitative Literacy ☑ Understands and applies number operations and properties
Big Ideas	° Creates a recursive model of a real life phenomena using NOW/NEXT and H(n)/H(n-1) notation ° Recognizes that situations in which the second difference is a constant can be modeled with quadratic functions ° Given 3 ordered pairs, finds the quadratic function passing through these points
Characteristics of Instructional Core	X Important core content that is evidence-based; Deep conceptual and procedural knowledge; Reflective teaching in a collaborative setting X Teaching for understanding; X Rigorous curriculum; Development of global perspective

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	x	Teaching through problem solving and inquiry	x	Relevant curriculum	x	Incorporation of current technology
	x	Student-centered classrooms	x	Assessment for learning		
Cognitive Domain		Remembering	x	Applying		Evaluating
	x	Understanding	x	Analyzing	x	Creating
Connections to Students' Lives		Isolated within discipline	x	Connected to other disciplines	x	Connected to student lives
	x	Connected within discipline		Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		
		Increasing access to print		Engaging students with texts		
	x	Involving students in discussion		Reading aloud in content areas		
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension		
		Writing to learn across content areas				
Class Time	2 class period(s)		90-110 total minutes			

Materials & Set-up:
Create enough copies of the student handout and student assignments for each student in the class. At one point in this lesson a system of three equations and three variables needs to be solved. Having a graphics calculator that can be projected to the class would be beneficial.

Teaching Tips:
An outline of the handshake lesson:

1. The students model the number of handshakes by shaking hands and counting
2. Model the number of handshakes recursively
   a. With Now/Next
   b. Using T(n) and T(n-1) and (n-1)
3. Model the number of handshakes explicitly
   a. Establish the fact that if the 2nd difference is a constant then the function is quadratic
   b. Create a system of equations to find the quadratic function
   c. Solve the system using matrices
   d. Use the solution of the system of equations to create an explicit formula
4. Verify that the number of handshakes counted by the students is consistent with the number of handshakes predicted by explicit and recursive formulas.

Comment [MB12]: ???

Teacher Tasks	Student Tasks
• Either have the students work in groups of four to model physically shaking hands with each other or have a group of four students model this for the class. • Create a discussion which builds the rationale for the Now/Next formula • Help the students make the connection	• Record the number of handshakes needed for n students to shake hands with everyone else exactly once for n=1,2,3,and 4 • Complete the appropriate part of the student handout dealing with the Now/Next formula.

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between the Now/Next formula and the T(n) and T(n-1) notation. • Create a chart with columns showing n (the number of people shaking hands), the number of handshakes, the first differences in the number of handshakes and the second differences in the number of handshakes. • Create a system of three equations and three variables which when solved will create a quadratic model for number of handshakes when n people are present • Guide a discussion in which the students solve the system of equations developed earlier. • Have the students verify that the explicit and recursive formulas give the same results.	• Complete the appropriate part of the student's handout dealing with the T(n) notation. • Contribute to the discussion creating the chart. • Contribute to the discussion that creates the system of equations. • Contribute to the discussion that creates a solution to the system of equations. • Verify the recursive and explicit formulas give the same predicted number of handshakes.
Differentiation Tasks	
Less-Than-Proficient Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction: quadratic, function notation, recursive formula, explicit formula • Less-than-proficient students may find the ANS+ feature on the calculator a nice way to compute the number of handshakes required. • Highly proficient learners can explore why the second differences of quadratic functions are constant and answer question 14.Since H(n) is a quadratic function, $H(n) = a n^2 + b n + c$. Because of the work done in problem #1, H(1) = 0, H(2) = 1, and H(3) = 3. Put these ideas together to create a system of three equations and three variables. Solve this system of equations and write the explicit formula for H(n). Show all of the work below.	

Assessment:

Formative Assessments:

- Student discussions and the results of the student handout will provide formative assessment of the students' learning.

Summative Assessments:

- The students' work on the assignment will provide an opportunity for summative assessment.

Reflect:

Comment [MB13]: Needs to be completed.

A goal of this lesson is to look at rates of change in table format noting the relationship between the patterns in the tables to the type of graph created. Connections to the first two activities of this unit can be made by looking at the data displayed in the Walk This Way Activity in table format. Students can notice very quickly that if the function is linear over an interval, the rate of change as shown by the first differences

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in table format or the slope in graph format is constant. Having the students make up a quadratic function and complete a table of values for that function over regular intervals similar to the chart shown in #1 on the student page will help them understand the idea that a constant second difference indicates a quadratic function.

Attachments:

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Student Page/Handout

Given the number of politicians who have recently visited the state of Iowa in the hopes of getting Iowans to caucus for them, there has been a great deal of handshaking going on across our great state. The purpose of this activity is to develop several methods that could be used to determine the number of handshakes that would occur if everyone in the room shook hands with everyone else in the room.
Form groups of three or four students and work together to act out situations where there are 1, 2, 3, and 4 students in a room. Determine the number of handshakes that would have to occur so that every student shook hands with every other student. Next complete the problems 1, 2 and 3 below.

1. Record the results of the handshake activity in the second column of the first four rows of the chart below.

# of people	# of handshakes	1st difference	2nd difference
1		XXXXXXXX	XXXXXXXX
2			XXXXXXXX
3			
4			
5			
6			
n			XXXXXXXX

2. Complete the first three rows of the column labeled "1st difference" by subtracting the number of handshakes for row 2 from the number of handshakes for row 1 and placing this difference in the column labeled 1st difference. Do the same for rows 3 and 4.

3. What is the relationship between the numbers in the "1st difference" column and the number of people?

4. Complete the "number of handshakes" column for rows 5 and 6 using the relationship noted in #3.

5. Compare the results of the first four questions with those of another group.

Be prepared to share your results with the class. As a class we'll discuss the following questions.

6. If NOW represents the number of handshakes for a given number of people, NEXT to represents the number of handshakes when one additional person is added to the room and n represents the given number of people, write an expression for NEXT in terms of NOW and n.

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7. Rewrite the NEXT/NOW equation using H(n) and H(n-1). In other words, if H(n)= the number of handshakes when n people are in the room and H(n-1)=the number of handshakes when n-1 people are in the room, write and equation for H(n) in terms of H(n-1) and n. (Since H(n) is expressed in terms of H(n-1), a previous value, this is a recursive formula.)

These recursive formulas provide a method of computing the number of handshakes. However, if a person would like to determine the number of handshakes when there are 100 people in the room, the number of handshakes for 99 people would need to be computed. To get this, the number of handshakes when 98 people are in the room would need to be computed. It would be nice if the number of handshakes could be computed directly from the number of people in the room. A function where the number of handshakes is determined only by the number of people in the room is an example of an explicit or closed-form formula.

8. Complete rows 3, 4, 5, and 6 for the column labeled "second difference".

If all of the computations are correct, the second differences should all be the same number. When the second differences are constant, this indicates the explicit function is quadratic. Likewise if the 3rd differences are constant, the function is cubic. In general when the nth differences are constant, the explicit formula relating the variables is an nth degree polynomial.

9. Using the values shown in the first and second columns of the table in #1, plot the ordered pairs (number of people, number of handshakes). Connect these points with a smooth curve.
10. What type of graph does h(n) appear to be?

N	H(n)
0	
1	
2	
3	
4	
5	
6	

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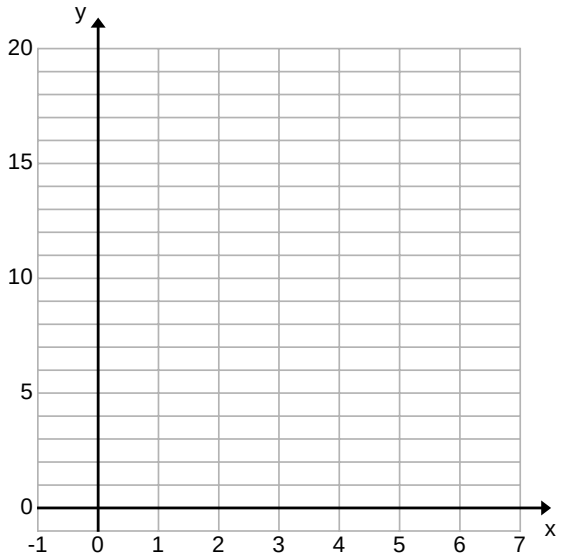

11. Complete the table using the function h(n)= $\frac{n(n-1)}{2}$. What do you notice about the values in the table compared to the plotted values?

12. What does n represent in this situation? What does n-1 represent? Why does dividing by 2 make sense in this situation? Explain why the expression for h(n) makes sense.

13. Is the graph of h the same type of graph you conjectured it would be in #10?
14. Use the results of question #4 to justify your choice for H(n) developed in #9.

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Comment [MB14]: The "second differences" stuff is nice, but not as relevant to future math courses. Recursion, of course, is a vital concept.

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Student Assignment:

A Recursive View of Skydiving

Skydiving is an exciting but dangerous sport. Many precautions are taken to ensure the safety of the skydivers. The basic fact underlying these precautions is that acceleration due to the force of gravity is 32 feet per second per second (written as 32 ft/sec^2). Thus, each second that the skydiver is falling, her speed increases by 32 ft/sec. (Throughout this investigation we ignore air resistance and other complicating factors; we focus only on the force of gravity.)

In this investigation, you should focus on these 3 questions:

What are recursive and explicit formulas for the total distance fallen by a skydiver after each second before her parachute opens?

What methods can you use to find these formulas?

How are quadratic functions involved and why?

To help answer these questions, consider the following table, which you will complete in the problems below.

A Skydiver's Speed and Distance Fallen Before the Parachute Opens

Time in seconds	**Speed** at time n	**Average Speed** *during* each second	**Distance Fallen** *during* each second D(n)	**Total Distance Fallen** *after* each second T(n)
0	0	0	0	0
1	32 ft/sec	16 ft/sec	16 ft	16 ft
2	64 ft/sec	48 ft/sec	48 ft	64 ft
3				
4				
:	:	:	:	:
n				

Comment [MB15]: This confuses me a bit.

Comment [JC16R15]: During the first second, the diver fell 16 ft, during the 2nd second the diver fell 48 ft.

Comment [MB17]: This confuses me a bit.

Comment [JC18R17]: The total distance the diver fell from the beginning of the dive to the end of the 1st second was 16 ft. The total distance the diver fell from the beginning of the dive to the end of the 2nd second was 64 ft.

1. Explain each entry in the row corresponding to Time = 1 sec in the table above. (The basis for computing all entries is the fact that acceleration due to gravity is 32 ft/sec^2.)

2. Explain each entry in the row corresponding to Time = 2 sec.

3. Complete the table for Time = 3 sec and Time = 4 sec. Compare your table entries to those of some of your classmates. Discuss and resolve any differences.

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Distance Fallen During Each Second – Now you will use the completed table to help find formulas for distance fallen. First, consider distance fallen during each second.

4. Find recursive and explicit formulas for D(n), Distance Fallen during the nth second, as follows. (For this problem, ignore the row in the table for Time = 0 sec.)

a. If NOW is the Distance Fallen during any given second and NEXT is the Distance Fallen during the next second, write an equation for NEXT in terms of NOW.

b. Rewrite the NEXT/NOW equation using D(n) and D(n-1).
That is, if D(n) = Distance Fallen during the nth second, and D(n-1) = Distance Fallen during the (n-1)st second, write an equation for D(n) in terms of D(n-1). (This is a ***recursive formula*** since D(n) is expressed in terms of a previous value, D(n-1).)

c. If D(n) = Distance Fallen during the nth second, write an equation for D(n) in terms of n. Explain how you got your equation and why it is correct.
(A formula like this, where D(n) is written as a function of n, is called an ***explicit* or *closed-form formula***.)

Total Distance Fallen After Each Second – The main goal of this investigation is to find formulas for T(n), the Total Distance Fallen after each second. You will use several methods to do this:

- use a general analysis
- use an arithmetic sequence
- use the method of Finite Differences.

Each of these methods is carried out in the next three problems.

5. Find formulas for Total Distance Fallen after each second, as follows. Let T(n) = Total Distance Fallen after the nth second.

a. As part of Problem 3, you computed T(3). Describe how you computed T(3).

b. Describe all the methods you can think of for how to compute T(n).

c. Write a formula for T(n) in terms of T(n-1) and D(n). (This is a *recursive formula* since T(n) is expressed in terms of the previous value, T(n-1).)

6. Find an explicit formula for T(n) by summing an arithmetic sequence, as follows. (This is optional – for those who have studied arithmetic sequences.)

a. One way that you may have described in Problem 5b for finding T(n) is to sum all the terms up to D(n) in the D(n) column. If you didn't already describe this in Problem 5b, explain here why this is a valid method for computing T(n).

b. You found in Problem 4c above that D(n) = D(n-1) + 32 ft (ignoring the row for Time = 0 sec). This formula shows that you add a constant, 32, each time to get the next value of D(n). Thus, the terms D(n) form an arithmetic sequence. Therefore, T(n) = the sum of the arithmetic sequence: D(1) + D(2) + … + D(n). Compute this sum to find an explicit formula for T(n) in terms of n.

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7. Another way to find an explicit formula for T(n) is to use a method called *finite differences*. Here's how it works.

a. Complete the three remaining entries in the bottom of the table below.

Finite Differences Table

n	T(n)	1st Differences (entry in the previous column) – (entry just above it)	2nd Differences (entry in the previous column) – (entry just above it)
1	16	----------	----------
2	64	64 - 16 = 48	----------
3	144	144 – 64 = 80	80 – 48 = 32
4	256	256 – 144 = 112	
5	400		

b. Describe the pattern in the 2nd differences column.

c. Now we apply a key fact: If the nth differences in a finite differences table are constant, then the formula for T(n) is an nth-degree polynomial. In this case, the 2nd differences are constant, so the formula for T(n) is a 2nd-degree polynomial, that is, the formula is quadratic. (Proving this key fact is not too hard, but it will take too long to do it now. Your teacher might give you guidance or references for the proof if you are interested.)

So we know that T(n) is quadratic and thus it looks like:

$$T(n) = an^2 + bn + c$$

Now we need to find the coefficients, a, b, and c. One way to find a, b, and c is to generate and solve a system of three linear equations. To help us do this, we know the value of T(n) for several values of n. Thus we get:

$$T(n) = an^2 + bn + c$$
$$n=1 \rightarrow \quad 16 = a + b + c$$
$$n=2 \rightarrow \quad 64 = 4a + 2b + c$$
$$n=3 \rightarrow \quad 144 = 9a + 3b + c$$

Explain the details of how these three equations are generated.

d. Now you need to solve this system of three linear equations. One way to do so is by using matrices. (For another method, see Problem 8 below.) To begin, a system of linear equations like this can be represented using matrices, as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 16 \\ 64 \\ 144 \end{bmatrix}$$

Explain where all the entries in the matrices come from, and why this matrix equation is equivalent to the linear system in Part c.

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e. You can solve this matrix equation by multiplying both sides of the equation on the left by the inverse of matrix $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}$.

This gives you: $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 16 \\ 64 \\ 144 \end{bmatrix}$.

Find this inverse matrix and carry out the multiplication to solve the matrix equation. (You may want to use your calculator to carry out these computations.) What are the values for a, b, and c?

f. Using the values for a, b, and c that you just found, what is the formula for T(n)?

g. Check the formula you found in Part f by evaluating it for some values of n, and verifying that you get the same values for T(n) as in the tables above.

8. Optional alternate method for solving the system of equations in Problem 7: You can also solve systems of linear equations like this using algebra without matrices. To do this, you need to combine and manipulate the three equations in Part c until you can solve for a, b, and c. The combining and manipulating is similar to what you do for a system of two linear equations, but more complicated since there are more equations. Try this method. Check that you get the same solution as in Part f. (For an example of how this is done in a similar problem, see *Mission Mathematics, Grades 9-12*, NCTM, p. 23.)

9. Summary

a. What are recursive and explicit formulas for T(n), the total distance fallen by the skydiver?

b. Describe the methods you used for finding the formulas in Part a.

c. The explicit formulas that you found for T(n) are quadratic functions. Compare patterns of the quadratic functions that you have worked with in this investigation to patterns of linear and exponential functions you have studied previously, as follows:

- Examine the list of values for T(n) shown in the table at the beginning of this investigation (the last column in the table on page 1). How is the pattern of change shown in the list of values for T(n) different from the pattern of change in tables for linear and exponential functions?
- How is the recursive formula you found for T(n) in Problem 5c different from recursive formulas for linear and exponential functions?
- How are the explicit formulas you found for T(n) in Problems 6 and 7 different from the explicit formulas for linear and exponential functions?

Comment [MB19]: Very nice

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ACTIVITY 4

Activity 4 Overview: Cliff Diver

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Iowa Core Curriculum Essential Skills/Concepts:	° *Essential Skills from the Iowa Core Curriculum…* *… in Problem Solving:* ☑ Builds new mathematical knowledge through problem solving ☑ Applies and adapts a variety of appropriate strategies to solve problems in mathematics and other contexts *… in Communication:* ☑ Organizes and consolidates his/her mathematical thinking through communication ☑ Communicates his/her mathematical thinking coherently and clearly to peers, teachers, and others ☑ Uses the language of mathematics to express mathematical ideas precisely *… in Reasoning and Proof:* ☑ Makes and investigates mathematical conjectures *… in Representation:* ☑ Creates and uses representations to organize, record, and communicate mathematical ideas ☑ Selects, applies, and translates among mathematical representations to solve problems ☑ Uses representations to model and interpret physical, social, and mathematical phenomena *… in Connections:* ☑ Recognizes and applies mathematics in contexts outside of mathematics ° *Essential Concepts from the Iowa Core Curriculum….* *….in Algebra:* ☑ Understands and applies functions ☑ Understands and applies equations and inequalities ☑ Understands and applies algebraic expressions ☑ Understands and applies rate of change ….in Geometry: *Understands and applies coordinates*
Big Ideas	° *Average rates of change for non-linear (quadratic) functions*

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	° *Estimates rates of change at a particular moment* ° *Determines intervals on which a function is increasing or decreasing*					
Characteristics of Instructional Core	X	Important core content that is evidence-based	X	Deep conceptual and procedural knowledge		Reflective teaching in a collaborative setting
	X	Teaching for understanding	X	Rigorous curriculum	X	Development of global perspective
	X	Teaching through problem solving and inquiry	X	Relevant curriculum	x	Incorporation of current technology
		Student-centered classrooms		Assessment for learning		
Cognitive Domain		Remembering	X	Applying	x	Evaluating
	X	Understanding	X	Analyzing	x	Creating
Connections to Students' Lives		Isolated within discipline	X	Connected to other disciplines	X	Connected to student lives
	X	Connected within discipline	X	Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		
		Increasing access to print	X	Engaging students with texts		
	X	Involving students in discussion	X	Reading aloud in content areas		
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension		
	X	Writing to learn across content areas				
Class Time	1 class period(s) 55 total minutes					

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Materials & Set-up:
Create a plan that will allow students to read the article found at the website:
http://www.signonsandiego.com/uniontrib/20060219/news_1t19diver.html .
Create a graph of h(t) so that slopes of secants segments can be drawn.

Teaching Tips:
This activity looks at rates of change using tables, graphs and function notation. It lays the ground work that can be built on to develop an understanding of the derivative.
As the cliff diving article is read, highlight the idea that author indicates the diver is traveling 55 mph as he or she enters the water. The purpose of this lesson is to come up with a method to verify this statement.
Throughout the discussion of average rates, reinforce the idea that the average velocity is the same as the slope of secant line segments drawn between two points on the h(t) curve. Also point out the relationship between the slope of the tangent line at a point and the instantaneous rate of change of the function at that point. A graphics calculator is needed to display h(t) for the students to see.

Teacher Tasks	Student Tasks
• Guide the students through the reading of the article about the cliff divers.	• Read the article about the cliff divers.
• With the students, create the graph of h(t)	• Complete the student handout.
• Discuss the average rate of change and how that changes as the interval over which the average is computed decreases.	• Complete the student handout.
• Throughout the class, encourage students to develop multiple expressions that could be used to compute the average rate of change	• Be thinking about alternative methods that could be used to approximate instantaneous rates of change.
Differentiation Tasks	
• For less-than-proficient students, relating the h(t) to an odometer on a car seems to make the concept of a position function more tangible. Therefore, by subtracting two odometer readings and dividing by the amount of time elapsed between readings, the average velocity can be computed. If taken over smaller and smaller time intervals this average does a better and better job of approximating the speedometer. • Highly proficient students can research the concept of limits and its relationship to instantaneous rates of change.	

Assessment:

Formative Assessments:
- Student discussions and the results of the student handout will provide formative assessment of the students' learning.

Summative Assessments:
- The students' work on the assignment will provide an opportunity for summative assessment.

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Attachments:

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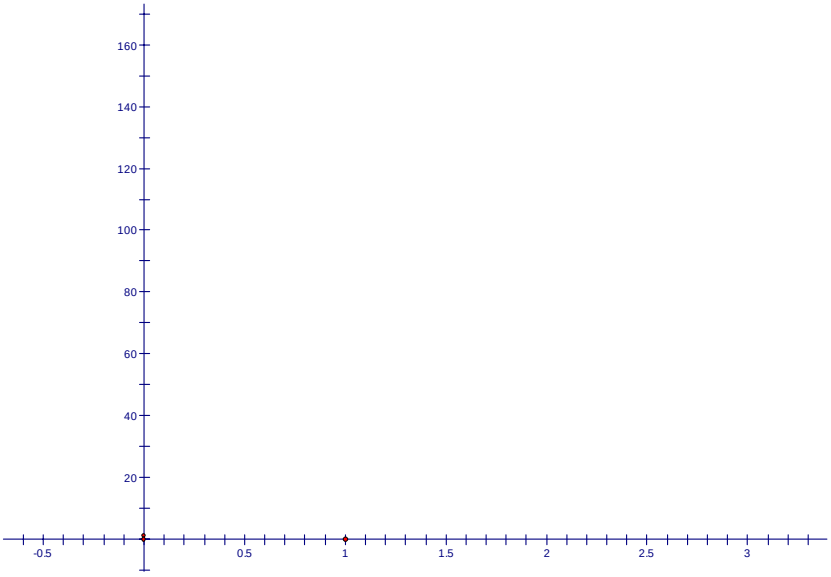
Student Page/Handout

The divers described in the article we read do not travel at the same rate throughout their descent to the water. However, it is possible to come up with a very good approximation for the rate of change in the height of the diver at any given moment. The average rate of change in the height of the diver over various time intervals is called the average velocity. The absolute value of the average velocity is called the average speed of the diver. Throughout this activity we'll investigate the average speed of the diver and the speed of the diver at various times throughout his fall.

Suppose the diver leaps up at a rate of 8 ft/sec from a cliff that is 110 feet above sea level. The height in feet of the diver above the water after t seconds can be modeled by the equation $h(t) = -16t^2 + 8t + 110$.

1. How long will it take the diver to reach the water?
2. Use a calculator to help create the graph of the function on the axis shown below. Use the x-axis for time and the y-axis for height.



3. When does the diver reach a maximum height? What is the maximum height?

4. Compute the average velocity of the diver from take off (when t=0) to maximum height.

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5. Draw a line segment connecting the point (0,110) to the point on the graph where the diver is at the maximum height. Find the slope of this line segment.
6. Compute the average velocity of the diver from maximum height to entry into the water.

7. Draw a line segment joining the point on the graph where the diver is at the maximum height to the point where the diver enters the water. Find the slope of this line segment.

8. How does the average velocity of the diver change throughout the diver's flight? How is this shown in the graph drawn in #1?

9. Determine the average speed of the diver from time that is 2 seconds after the diver jumps to the time the diver enters the water.

10. The answers to questions 6 and 9 indicate the average speed of the diver over two different intervals. Which is a better approximation for the speed when the diver enters the water?

11. How could the velocity at a particular instant be approximated? With a partner discuss how to approximate the speed of the diver as he enters the water.

Compare the results of your discussion with the process outlined in #12.

12. Determine the average speed of the diver during these time intervals during his flight.
    a. From t=.25 to t=2.88.
    b. From t=1.88 to t=2.88
    c. From t=2.50 to t=2.88
    d. From t=2.87 to t=2.88
    e. What do the answers to parts a through d indicate about the divers' speed when he hits the water?

13. Think about the speed of the diver at some other points throughout his flight. Develop some other methods of approximating each of the following:
    a. The diver's speed at exactly 0.25 second into the dive

    b. The diver's speed at exactly 1 second into the dive

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c. The diver's speed at exactly 2 seconds into the dive

14. Let's see if there is a formula that will make these calculations less tedious. Consider the function S(t) = $\frac{h(t+0.01)-h(t)}{0.01}$. Let h(t) represent the height of the diver after t seconds.

a. Complete the table shown below

t	h(t)	S(t) = $\frac{h(t+0.01)-h(t)}{0.01}$.
0		
0.25		
1		
2		
2.8		
2.88		

b. Compare the answers for t=0.25, t=1 and t=2 to your answers for 12a, 12b and 12c. What do you notice?

c. What are the units of measure for S(t)? Convert S(2.88) to miles per hour. How fast did the author of the article say the diver was traveling when he hit the water?

d. Explain why S(t) gives a good approximation for the velocity of the diver at a given moment.

e. How could the approximations for S(t) be improved?

f. How could S(t) be altered to approximate rates of change for other functions?

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Assignment

1. When a function is increasing what is the sign for the rate of change?

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2. When a function is decreasing what is the sign for the rate of change?

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3. Using f(x+h), f(x) and h where h is some amount of time, describe how the average rate of change in a function is computed.

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4. Describe how the rate of change at a particular point can be approximated.

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Comment [MB20]: YES! Has this been field-tested? If it works, this is a great pre-calc thing for my entering students to have done.

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ACTIVITY 5

Activity 5 Overview: The Fish Population

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Iowa Core Curriculum Essential Skills/Concepts:	
	° *Essential Skills from the Iowa Core Curriculum…* *… in Problem Solving:* ☑ Builds new mathematical knowledge through problem solving ☑ Applies and adapts a variety of appropriate strategies to solve problems in mathematics and other contexts *… in Communication:* ☑ Organizes and consolidates his/her mathematical thinking through communication ☑ Communicates his/her mathematical thinking coherently and clearly to peers, teachers, and others ☑ Uses the language of mathematics to express mathematical ideas precisely *… in Representation:* ☑ Creates and uses representations to organize, record, and communicate mathematical ideas ☑ Selects, applies, and translates among mathematical representations to solve problems ☑ Uses representations to model and interpret physical, social, and mathematical phenomena *… in Connections:* ☑ Recognizes and applies mathematics in contexts outside of mathematics ° *Essential Concepts from the Iowa Core Curriculum….* *….in Algebra:* ☑ Understands and applies functions ☑ Understands and applies equations and inequalities ☑ Understands and applies algebraic expressions ☑ Understands and applies rate of change *….in Geometry:* ☑ Understands and applies coordinates

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Big Ideas	° *Estimates rates of change at a particular moment* ° *Calculate the instantaneous rate of change using the derivative formula and the graphing calculator* ° *Gives real world interpretation to rates of change in a graph* ° *Interprets shape and meaning of the logistic curve*					
Characteristics of Instructional Core	X	Important core content that is evidence-based	X	Deep conceptual and procedural knowledge		Reflective teaching in a collaborative setting
	X	Teaching for understanding	X	Rigorous curriculum		Development of global perspective
	X	Teaching through problem solving and inquiry	X	Relevant curriculum	x	Incorporation of current technology
		Student-centered classrooms		Assessment for learning		
Cognitive Domain		Remembering	X	Applying	x	Evaluating
	X	Understanding	X	Analyzing		Creating
Connections to Students' Lives		Isolated within discipline	X	Connected to other disciplines	X	Connected to student lives
	X	Connected within discipline	X	Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		
		Increasing access to print	X	Engaging students with texts		
	X	Involving students in discussion		Reading aloud in content areas		
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension		
	X	Writing to learn across content areas				
Class Time	1 class period(s)	55 total minutes				

Materials & Set-up:
The teacher will need a projection of the graph of the fish population, either a graphing calculator or simply a transparency.
Using a graphing calculator projection could allow for more flexibility in zooming and adjusting the parameters of the logistic equation
Students will each need a copy of the worksheet which has the graph, function, and assessment questions on it.

Teaching Tips:

This activity focuses on the concept of rate of change by looking at tables, graphs and the difference function for average rates of change much like the previous activity. Students in this activity also begin to compare rates of change over different time intervals.
The teacher should introduce the problem and show the graph to the students. The class should discuss why a population problem like this may be realistically described by the graph shown. Most likely students have not seen a logistic curve such as this before, so that language can be introduced and the formula for it discussed. The main purpose of the lesson is for students to be able to estimate and then calculate instantaneous rates of change, and describe them in context of the fish population over time.

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Teacher Tasks	Student Tasks
• Introduce the situation of the fish population problem • Lead a discussion of the shape of the graph, important points and possible explanations of the shape, and its relation to the fish population • Check for correct entry of the logistic equation into students calculators as Y1, paying particular attention to the denominator • Discuss the method for finding instantaneous rate of change using the graphing calculator, comparing it to the method used previously in The Diver Problem • Check for understanding of the instantaneous rate of change formula that students will enter into Y2 on their calculators, especially getting Y1 entered into the calculator and understanding its purpose.	• Examine the graph and participate in discussion of the shape and why the shape may make sense for this situation • Enter the logistic equation into their graphing calculators and create a table of the fish population for the first 20 years • Complete student handout

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Differentiation Tasks

- For highly proficient students, an extension of the problem could be done with the logistic equation. Students could investigate how and where this form of equation used. Can students identify what effect the parameters have on the graph of the equation? What does each parameter affect in context of a changing population? $f(x) = \frac{a}{1+b(c)^x}$ represents a simplified version of the logistic equation with the parameters a, b, and c. In the Fish Population problem, a = 100, b = 20, and c = 0.5. Students could also investigate the logistic regression feature of the TI-84 calculators, given a set of population data.
- Highly proficient students could compare How is Y_2 in this activity is similar to S(t) from #14 in Activity #4?
- Less than proficient students may need more assistance, particularly with the formula for instantaneous rate of change entered into Y2 of the calculator on the student work page. They may need a series of examples of secant lines that are spread further away from the desired point than the stated 0.1 on either side. This may make the idea of "zooming" in on a specific point more accessible.

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Assessment:

Formative Assessments:

- Student discussions and the results of the student handout will provide formative assessment of the students' learning.

Summative Assessments:

- The students' work on the assignment will provide an opportunity for summative assessment.

Attachments:

Student Page/Handout

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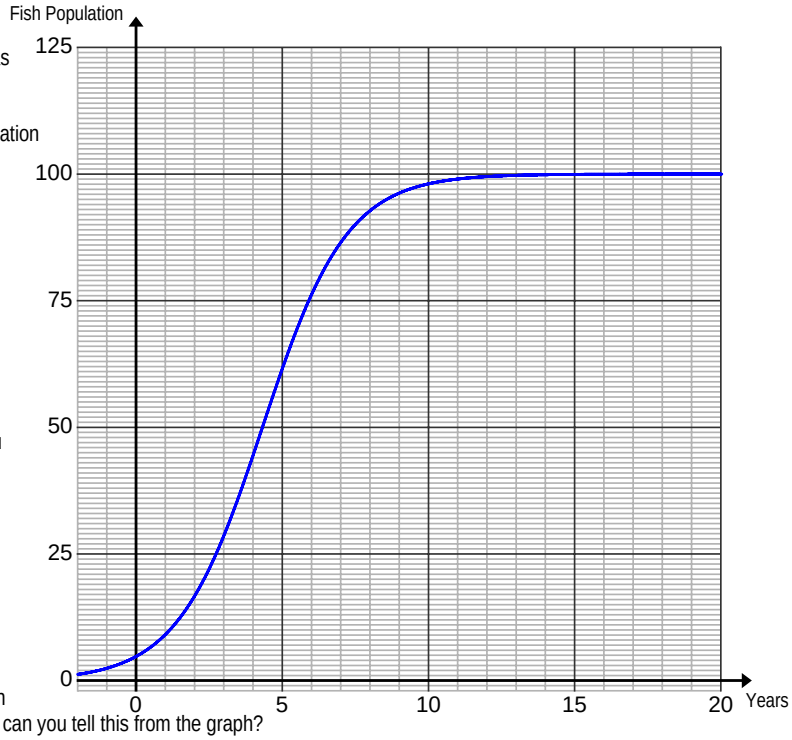
The Fish Population

The graph to the right shows the pattern of growth in the fish population of a lake that was stocked in 1985.



1. Describe what happens to the fish population over the 20 year period from 1985 – 2005.

2. Why do you think that the graph has the shape that it does? What factors do you think would cause such a shape in the population graph?

3. Use the graph to estimate how many fish were initially stocked into the lake. How can you tell this from the graph?

4. At what point in time does it appear that the fish population was increasing the most rapidly as time passed? How can you tell this from the graph?

5. This graph is modeled well by the function rule $P(x) = \dfrac{100}{1+20\left(0.5^{x}\right)}$, where x represents the number of years after 1985. Functions of this form are called *logistic* functions. Enter this function into Y_1 of your calculator and use it to fill in the population table on the back side of this worksheet.

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Year, *x*	Population
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

6. In Y_1 of your calculator, you entered the function rule for the fish population, $P(x) = \dfrac{100}{1+20(0.5^x)}$. In Y_2 of your calculator, enter the following rule:

$$Y_2 = \frac{Y_1(x+0.1) - Y_1(x-0.1)}{0.2}.$$

7. What does $Y_1(x+0.1)$ calculate in terms of the fish population?

8. What does $Y_1(x-0.1)$ calculate in terms of the fish population?

9. Why is $Y_1(x+0.1) - Y_1(x-0.1)$ divided by .2?

10. What does this new function you entered into Y_2 do?

11. How could you alter the function you entered in Y_2 to make it even more accurate?

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12. At what point in time is the fish population increasing the most rapidly? How does your answer now compare to your visual estimate from problem #2? How rapidly is the fish population changing at this point in time?

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13. Describe how the fish population is changing before the time you identified in problem #8. How is the fish population changing after that time? A point that divides rates of change like this is called an *inflection* point.

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14. Using correct labels give the rate of change for the fish population at 2,4,6, and 8 years. Describe how the rate of change is shown in the graph at those times.

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15. Describe what happens to the fish population between 10 and 20 years. Why do you think this happens? What is the rate of change in the fish population at 10, 15, and 20 years?

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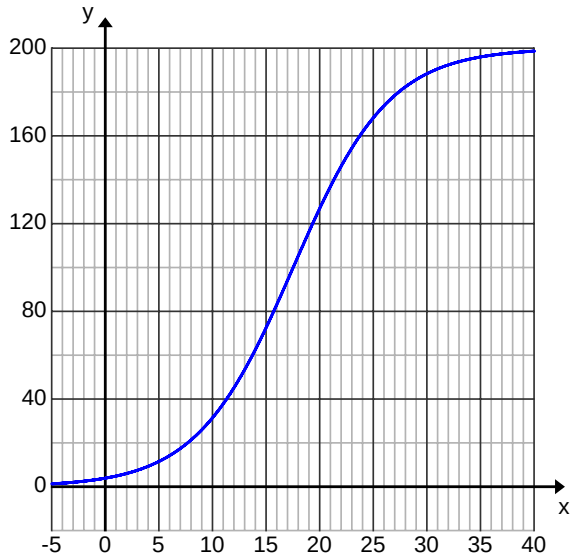
Assignment

Comment [MB21]: Very nice

Comment [MB22]:

Comment [MB23]:

For the logistic graph shown, answer the following:



1. At what level does this population appear to level out?

2. What is the approximate rate of change in population at year 40?

3. At what point in time does it appear that the rate of change in the population is greatest? Explain how you know based on the graph.

4. The population graph shown is represented by $f(x) = \dfrac{200}{1+50(0.8)^x}$. Using this function, show how you calculate the rate of change in the population at the year you identified in problem #3 above.

5. Calculate the rate of change in the population two years <u>*before*</u> the year you identified in problem #3. Also calculate the rate of change for two years <u>*after*</u> the year in #3. Do these rates of change help to confirm your answer in #3? Why or why not?

Comment [MB24]: Very nice.

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