![IOWA CORE CURRICULUM]

Subject: Math II
Developer Name:

Unit Title: Introduction to Transformations
Location:

Course Description:
This is designed for an integrated math two class. It could be used in any course as an introduction to transformations from pre-algebra to algebra two.

Unit Context:
This unit is an introduction to the rigid transformations: translation, rotation, and reflection. Dilations are included as a non-rigid transformation. Connecting the strands of mathematics, the students explore transformations on the coordinate and synthetic plane, transformations of graphs of functions, and of a set of data. The unit culminates in the students creating an animation. The prerequisite knowledge required for the unit is minimal. It would be helpful, yet not required, for students to know measures of central tendency and a measure of variability such as standard deviation. In addition, it would be helpful, but not required to have graphed basic functions.

Length of Unit:
Approximately 13 days of 50 minute class periods.

Iowa Tests Core Content Standards and Benchmarks
List all of the Iowa Tests Core Content Standards and Benchmarks that are addressed in this unit.

Core Content Standards and Benchmarks
• Students can understand and apply concepts of geometry and measurement.   • Students can understand and apply concepts and procedures of algebra.   • Students can understand and apply concepts in probability and statistics.

ICC Essential Concepts, Skills, and Characteristics
List all of the Iowa Core Curriculum Essential Skills, Concepts, and Characteristics that are addressed in this unit.

Concepts
• Understands and applies functions   • Understands and applies rates of change   • Understands and applies transformations   • Understands and applies descriptive statistics   • Understands and applies coordinates

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Skills

- Communicates his/her mathematical thinking coherently and clearly to peers, teachers, and others
- Uses the language of mathematics to express mathematical ideas precisely
- Uses representations to model and interpret physical, social, and mathematical phenomena
- Selects, applies, and translates among mathematical representations to solve problems
- Creates and uses representations to organize, record, and communicate mathematical ideas
- Recognizes and uses connections among mathematical ideas and how they build on one another to produce a coherent whole
- Recognizes and applies mathematics in contexts outside of mathematics

Characteristics

- Teaching for Understanding
- Problem-Based Instructional Tasks
- Mathematical Modeling
- Deep Conceptual and Procedural Knowledge
- Rigor and Relevance
- Effective Use of Technology
- Integrated Content

Big Ideas Addressed in Each Activity

Activity 1: Introduction to Transformations

° Geometric transformations (translation, dilation, reflection, and rotation) are found in nature, graphic design and art.
° Definitions can be created with examples and non-examples.

Activity 2: Translations and Reflections

° Creation of translations require an understanding of vectors.
° Creation of reflections require an understanding of perpendicular bisectors.
° Translations and reflections are identified in nature, graphic design and art by their key features.
° Students use technology as a tool to make sense of transformations and to create transformations.

Activity 3: Rotations and Dilations

° Creation of rotations require a center and angle of rotation.
° Creation of dilations require a center and magnification factor.
° Rotations and Dilations are identified in nature, graphic design and art by their key features.
° Students use technology as a tool to make sense of transformations and to create transformations.
° There are transformations that preserve shape and/or distance.

Activity 4: Transformations on the Coordinate Plane

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° Coordinates can be used to describe points, lines, and other two- and three-dimensional figures. Transformations of these objects can be described using coordinate rules.

° Transformation can be identified, created, described, and justified using multiple representations. Students should be able to find and describe an image under a given transformation and/or composition of transformations. Students should also be able to identify the transformations that produced a given image. Transformations should be represented algebraically (using coordinate rules, matrices, vectors, equations), and those representations should be used to analyze and reason about transformations.

Activity 5: Representing Transformations using Matrices

° Transformation can be identified, created, described, and justified using multiple representations. Students should be able to find and describe an image under a given transformation and/or composition of transformations. Students should also be able to identify the transformations that produced a given image. Transformations should be represented algebraically (using coordinate rules, matrices, vectors, equations), and those representations should be used to analyze and reason about transformations.

Activity 6: Transformations of Functions

° Transformations such as reflection, translation and dilation are performed in the same way regardless of the family of function.

° Function families have a parent function and transformations of that parent function. Quick sketches of functions can be done by understanding the transformations
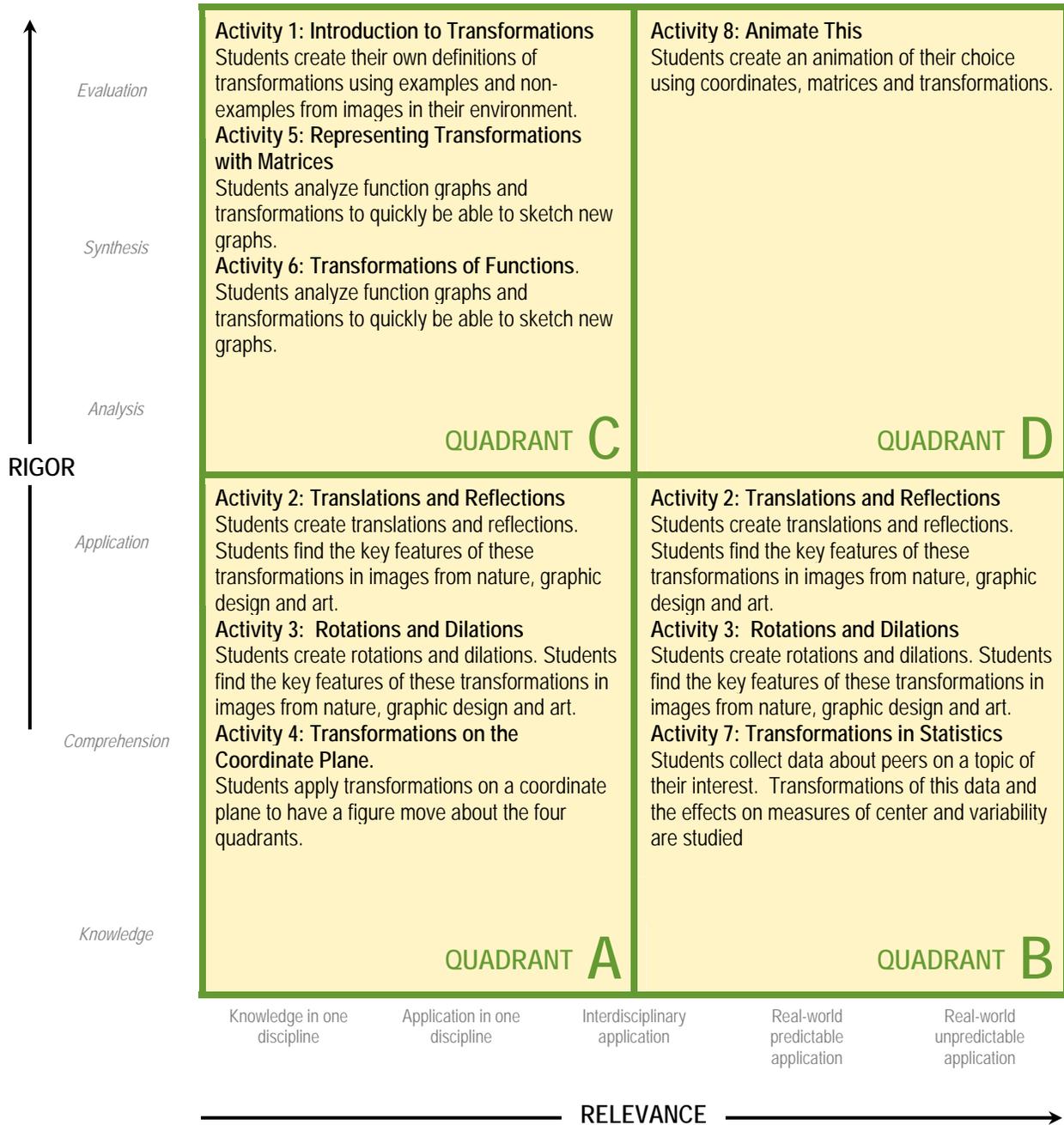
Activity 7: Transformations in Statistics

° Analysis of data should include the choice of appropriate representation, the study of measures of center and variability, transformations of univariate data, shape of distributions, outliers, regression, and correlation.

Activity 8: Animate This!

° Animations can be created by using coordinates, matrices and transformations. It is important to have a visual definition of each transformation in order to put a sequence of transformations together in a logical way.

Rigor & Relevance Plot: *Transformations*

RIGOR

Evaluation

Synthesis

Analysis

Application

Comprehension

Knowledge

Activity 1: Introduction to Transformations
Students create their own definitions of transformations using examples and non-examples from images in their environment.
Activity 5: Representing Transformations with Matrices
Students analyze function graphs and transformations to quickly be able to sketch new graphs.
Activity 6: Transformations of Functions.
Students analyze function graphs and transformations to quickly be able to sketch new graphs.

QUADRANT **C**

Activity 8: Animate This
Students create an animation of their choice using coordinates, matrices and transformations.

QUADRANT **D**

Activity 2: Translations and Reflections
Students create translations and reflections. Students find the key features of these transformations in images from nature, graphic design and art.
Activity 3: Rotations and Dilations
Students create rotations and dilations. Students find the key features of these transformations in images from nature, graphic design and art.
Activity 4: Transformations on the Coordinate Plane.
Students apply transformations on a coordinate plane to have a figure move about the four quadrants.

QUADRANT **A**

Activity 2: Translations and Reflections
Students create translations and reflections. Students find the key features of these transformations in images from nature, graphic design and art.
Activity 3: Rotations and Dilations
Students create rotations and dilations. Students find the key features of these transformations in images from nature, graphic design and art.
Activity 7: Transformations in Statistics
Students collect data about peers on a topic of their interest. Transformations of this data and the effects on measures of center and variability are studied

QUADRANT **B**

| Knowledge in one discipline | Application in one discipline | Interdisciplinary application | Real-world predictable application | Real-world unpredictable application |

RELEVANCE

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ACTIVITY 1

Activity 1 Overview: *Introduction to Transformations*

Iowa Core Curriculum Essential Skills/Concepts:	° Understands and applies transformations ° Communicates his/her mathematical thinking coherently and clearly to peers, teachers, and others ° Recognizes and applies mathematics in contexts outside of mathematics				
Big Ideas	° Geometric transformations (translation, dilation, reflection, and rotation) are found in nature, graphic design and art. ° Definitions can be created with examples and non-examples.				
Characteristics of Instructional Core	x	Encourages collaboration in learning	x	Teaches for understanding		Develops global perspectives
	x	Student centered		Develops conceptual and/or procedural knowledge	x	Provides authentic learning
	x	Teaches through problem solving & inquiry		Assesses for learning		Incorporates current technology
Cognitive Domain		Remembering		Applying		Evaluating
		Understanding	x	Analyzing		Creating
Connections to Students' Lives		Isolated within discipline		Connected to other disciplines	x	Connected to student lives
		Connected within discipline		Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		
		Increasing access to print		Engaging students with texts		
		Involving students in discussion		Reading aloud in content areas		
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension		
		Writing to learn across content areas				
Class Time	1 class period(s) 50 total minutes				

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Materials & Set-up:
- o Set of transformation cards (see attachment) or make a different set of transformation cards.
- o May want a way to project the images of the cards to facilitate the discussion.
- o Create groups of 3 to 4 students

Teaching Tips:
Depending on class dynamics and student ability, this task can be open or more structured. It has worked in my classroom to give them a cut out and have student groups share their ideas with other student groups before having a full class discussion. The discussion can be rich with connections to prior knowledge and to other strands of mathematics. The intent is to build informal understanding of the transformations for the students and to serve as pre-assessment for the teacher.

Teacher Tasks	Student Tasks
• Prior to activity create set of transformation cards. • Facilitate launch discussion, listening for student understanding of symmetry. The purpose of the launch is not to have completely accurate knowledge. The "explore" portion will help facilitate this. • Let student groups share their ideas about classification with a focus on why they classified a shape in a particular category. • Introduce more formal mathematical language gradually. • Facilitates the summary at the end of the task.	• Participate in the full class discussion of the launch. • Actively engage in the classification activity. • Share classifications with other groups or whole group. • Begin developing more formal mathematical language. • Create individual definitions for the transformations.
Differentiation Tasks	
• This task is typically open enough that most students do not need much differentiation for the less-than-proficient. Students may need mini-scaffolds when creating notes during the summarization. • For the highly proficient students, there may be complex figures which include visuals of many of the transformations.	

Assessment:

Formative Assessments:
- o The discussion following the categorizing will be a key moment to understand student thinking. In addition, the definitions students all create will give the teacher an idea of the depth of knowledge for each student.

Summative Assessments
- o The check for understanding will provide small summative assessment.

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Reflect:
Students engaged in the task, but really struggled with the cards that could be classified in more than one category. I may rework this in the future. My attempt to interject formal mathematical language was not successful with the majority of the class. Student definitions for the most part were informal, but meaningful to students. I do believe that students began to see "transformations" in their world.

Possible Cards to Use in Activity

#1

Reflection

#2
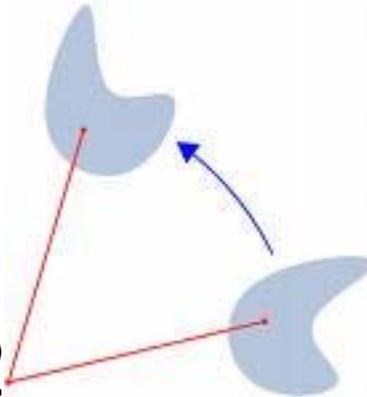
Rotation

#3

Reflection or Rotation

#4
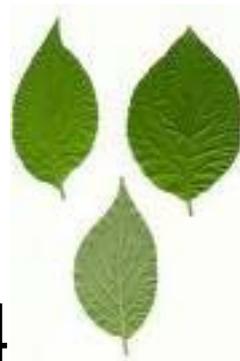
Reflection (some students argued for dilation)

Superman
Theme
(Music)

#5

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

#6
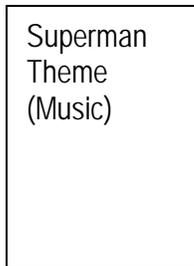
Translation

Translation

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

#7
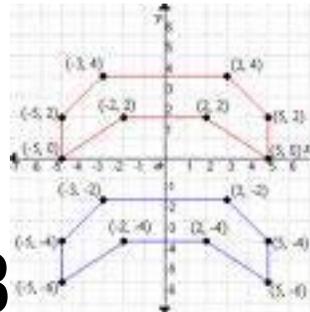
#8

Reflection or Rotation

Reflection or translation
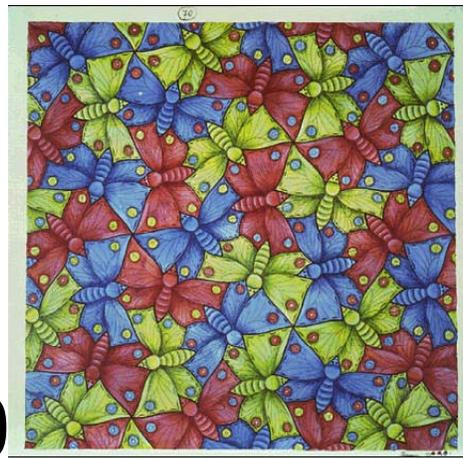
#9

#10

Dilation

Rotation

#11
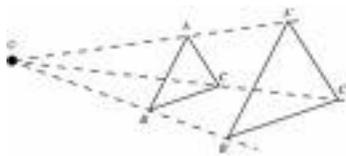
QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

#12

Dilation

Dilation

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#13
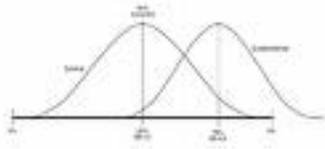
translation

#14

Rotation

#15

#16

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Activity 1 Student Page/Handout

Introduction to Transformations

Launch

1) What do you already know about symmetry?

2) What are some symmetries you have studied in the past?

Exploration

Symmetry is the preservation of form and configuration across a point, a line, or a plane. In informal terms, symmetry is the ability to take a shape and match it exactly to another shape. The techniques that are used to "take a shape and match it exactly to another" are called *transformations* and include translations, reflections, and rotations. Another transformation called a dilation maintains shape but not size.

Your group has been given a set of 16 cards. Create *four* different classifications or groups in which you can place the cards based on common symmetries and/or transformations. Cards #1-4 are each an example of a different classification. With your class decide where the remaining cards should be placed. Some cards may be able to be placed in more than one category. Be ready to share with the class how you created your groups.

Summarize

a) What are the defining characteristics of each group?

b) With your group, combine your comments about the characteristics of translations, reflections, and rotations. Be ready to share with the group your understanding of these transformations. Know that you are *not* expected to have formal definitions of each transformation, just your understanding in your own words. In future activities, you will explore each of these transformations.

Assignment: Check for Understanding

Look at your environment in terms of transformations and symmetry. Where do you see or experience them? Record at least five examples. Be sure to explain what transformation/symmetry you see. Be ready to share them with the class.

ACTIVITY 2

Activity 2 Overview: Translations and Reflections

<table>
<tr>
<td>Iowa Core Curriculum
Essential Skills/Concepts:</td>
<td colspan="9">
° Understands and applies transformations
° Uses the language of mathematics to express mathematical ideas precisely
° Uses representations to model and interpret physical, social, and mathematical phenomena
° Recognizes and uses connections among mathematical ideas and how they build on one another to produce a coherent whole
° Recognizes and applies mathematics in contexts outside of mathematics
</td>
</tr>
<tr>
<td>Big Ideas</td>
<td colspan="9">
° Creation of translations require an understanding of vectors.
° Creation of reflections require an understanding of perpendicular bisectors.
° Translations and reflections are identified in nature, graphic design and art by their key features.
° Students use technology as a tool to make sense of transformations and to create transformations.
</td>
</tr>
<tr>
<td rowspan="4">Characteristics of Instructional Core</td>
<td>x</td><td>Important core content that is evidence-based</td>
<td></td><td>Deep conceptual and procedural knowledge</td>
<td>x</td><td>Reflective teaching in a collaborative setting</td>
</tr>
<tr>
<td>x</td><td>Teaching for understanding</td>
<td></td><td>Rigorous curriculum</td>
<td></td><td>Development of global perspective</td>
</tr>
<tr>
<td>x</td><td>Teaching through problem solving and inquiry</td>
<td>x</td><td>Relevant curriculum</td>
<td>x</td><td>Incorporation of current technology</td>
</tr>
<tr>
<td>x</td><td>Student-centered classrooms</td>
<td>x</td><td>Assessment for learning</td>
<td></td><td></td>
</tr>
<tr>
<td rowspan="2">Cognitive Domain</td>
<td>x</td><td>Remembering</td>
<td>x</td><td>Applying</td>
<td></td><td>Evaluating</td>
</tr>
<tr>
<td>x</td><td>Understanding</td>
<td></td><td>Analyzing</td>
<td></td><td>Creating</td>
</tr>
<tr>
<td rowspan="2">Connections to Students' Lives</td>
<td></td><td>Isolated within discipline</td>
<td></td><td>Connected to other disciplines</td>
<td>x</td><td>Connected to student lives</td>
</tr>
<tr>
<td>x</td><td>Connected within discipline</td>
<td></td><td>Has value beyond school purposes</td>
<td></td><td></td>
</tr>
<tr>
<td rowspan="5">Support for Literacy</td>
<td></td><td colspan="3">Using the literacy process for inquiry</td>
<td></td><td>Increasing reading volume</td>
</tr>
<tr>
<td></td><td colspan="3">Increasing access to print</td>
<td></td><td>Engaging students with texts</td>
</tr>
<tr>
<td></td><td colspan="3">Involving students in discussion</td>
<td></td><td>Reading aloud in content areas</td>
</tr>
<tr>
<td></td><td colspan="3">Increasing reading fluency</td>
<td></td><td rowspan="2">Explicitly instructing in vocabulary and comprehension</td>
</tr>
<tr>
<td></td><td colspan="3">Writing to learn across content areas</td>
<td></td></tr>
<tr>
<td>Class Time</td>
<td colspan="9">1 class period(s)   50 total minutes</td>
</tr>
</table>

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Materials & Set-up:
Ideally all students would have a computer with an internet connection. If not, this can be done with one computer as a demonstration. Students will also need a protractor and a ruler. A digital camera for students would be ideal, but is not necessary. Also, Geometer's Sketchpad can be used during the activity.

Teaching Tips:
The purpose of this investigation is to introduce students to the different uses of translations and reflections. It also intends to formalize the definition of translation and reflection. Activity 3 is parallel to this activity, but focuses on rotations and dilations.

Teacher Tasks	Student Tasks
• Facilitate discussion of launch focusing on more specific mathematics included in a translation. • Facilitate student learning through the translation questions. • May want to have a mini-summary after the translation questions • Facilitate student learning through the reflection questions. • Facilitate the summary at the end of the task	• During launch add suggestions of more specific mathematics needed for a translation. • Engage in questions on student handout regarding translations. Actively question and explore to understand the mathematics. • Take notes during a mini-summary. • Engage in questions on student handout regarding reflections. Actively question and explore to understand the mathematics. • Take notes and contribute to the summarize.
Differentiation Tasks	

Less-Than-Proficient
Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction:
- vocabulary: isometry, vector, line of symmetry, perpendicular bisector
- skills/concepts: constructing a translation and a reflection by hand and using the computer software.

Supplemental instruction suggestions:
- Strategic partnerships for group work
- Create scaffolded notes for summary
- Have a Geometer's Sketchpad quick key stroke guide for students.

Highly Proficient
Supplemental instruction suggestions:
- Pretesting may suggest the need for compacting of some activities
- Have students work at an accelerated pace and provide questions about composition of two translations and two rotations.

Intensive instructional suggestions for students who are highly proficient
- Pretesting may show a clear need for an alternative unit using college-level geometry materials.

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Assessment:

Formative Assessments:
- o Listen carefully to student responses to key questions in the student handout:
  What do you think are some characteristics of translations?
  What do you think are some characteristics of reflections?
- o The summarize should be used to listen to student understanding and make instructional decisions on the next moves depending on the depth of understanding.

Summative Assessments:
- o The "Check for Understanding" can be used as a more summative assessment. Also Activity 8 will serve as a part of a summative assessment for the concepts in this unit.

Reflect:

The students seemed to enjoy the lesson and the different resources used throughout the activity. Not all students immediately saw a connection between the transformations in the different strands of mathematics. This needed to be facilitated much more by the instructor than anticipated. By the end of the investigation students seemed to have a good understanding of translation beyond "it moves an object" and a better understanding of a reflection. Completing a reflection by hand was not as intuitive as I believed it would be after the opportunities presented in the lesson.

Attachments:

Suggested Solutions to Activity 2 Student Handout
Part One: Translations
1. a) The image shape moves in the plane and is always the same distance from the original shape.
   b) The arrow gets smaller or larger and can change directions. The preimage and image shape are the same distance apart as the arrow.
   c) Has a preimage and image that is a certain distance apart and direction. Student responses may vary in the use of formal language.
2. a) The blue graph moves up and down (may say that the graph gets skinnier and wider)
   b) The difference between f(x) and g(x) always has an absolute value of 5.
   c) Answers will vary for students
3. Students can do this on a copy of the image or upload a digital image into sketchpad and "find" the translation vector.
4. All the distances are the same.
5. Drawing should show a new quadrilateral in a parallel direction to the translation vector. All corresponding vertices for the preimage and image quadrilateral should be the same distance apart which is the length of the translation vector.

Part Two: Reflections
6. a) The images are "connected" in some way. If you would mentally fold on the line the images would match up. Again students will have a variety of responses.
   b) The line moves and the figures move in a similar pattern to the line. Students may have a variety of responses.

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c) The pre-image and image figures are always equidistant from the line. The orientation of the figures change as the line is moved.

d) Reflections have a line of symmetry. The pre-image and image are always congruent. The dotted line that connects them is always perpendicular to the line of reflection. There could be more observations.

7. a) The parabolas are "opposite" of each other.

b) The values of the functions are opposites of each other.

c) Answers will vary with students.

8. Answers will depend on student work.

9.
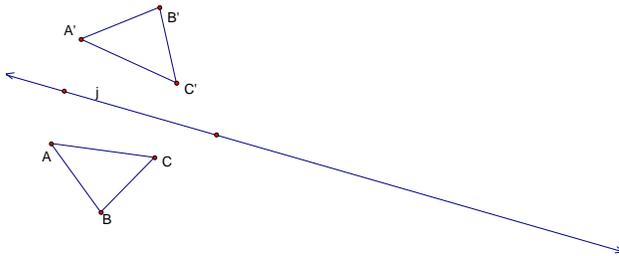
Summarize

Translations are isometries (rigid transformations) that are defined by vectors (magnitude and direction). A translation moves every point of a shape a constant distance in a specified direction.

Reflections are isometries that are determined by a line of reflection which is the perpendicular bisector of the segment connecting a point and its reflected image.

Activity 2 Student Page/Handout

Translations and Reflections

There are several different types of symmetry, but in each type of symmetry, characteristics such as angles, side lengths, distances, shapes, and sizes are maintained. Each of the transformations mentioned above produce a different type of symmetry. We will now discuss each transformation and its associated symmetry.[1]

Launch

One student chose the picture below and had grouped it under reflectional symmetry.

Figure 1

Think about how you could more precisely classify some of the examples from each group in Activity 1. What features or characteristics would you need to identify?

Exploration

Part One: Translations

1. Go to the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) and choose 9-12 Geometry and then choose Transformations—Translations. Finally, click on instructions. Shown below is one possible screen shot.

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QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

*Taken from http://nlvm.usu.edu/en/nav/category_g_4_t_3.html

a. Click on the shape and drag it. What happens?

b. Click on the arrow and drag it. What happens?

c. What do you think are some characteristics of translations?

2. Now consider a different look at translations Go to the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) and choose 9-12 Algebra and then, choose Grapher. Click on the Function tab and type $f(x)=x^2$. (Note: The blue boxes about the equation to help format.) Click on graph. Then, click on g(x) and type $g(x)= x^2 +a$. Then, click graph. Click on the parameter tab and use the slider for a. Shown below is one possible screen shot.

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

a. What happens to the graph of the function g(x) when you use the slider for *a*?
b. Slide the parameter to a=5. Now be sure the trace box is checked at the bottom right hand corner. Move the trace slider at the bottom of the screen. What do you notice about the difference between values of g(x) and f(x) for any x value?

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c. How is this the same or different to the first example with the square?

3. Using the image gallery that was created from Activity 1, choose an image that you believe has translational symmetry. Find the translation vector and place it on the picture. This can be done on paper or pasted into Sketchpad. Be specific about how you know that this is the line of symmetry.

4. In figure 2, $\triangle ABC$ was translated to $\triangle A'B'C'$ by the given translation vector. Find the distance between AA′, BB′ and CC′. Also measure the length of the given translation vector. How do these all compare?

Translation vector
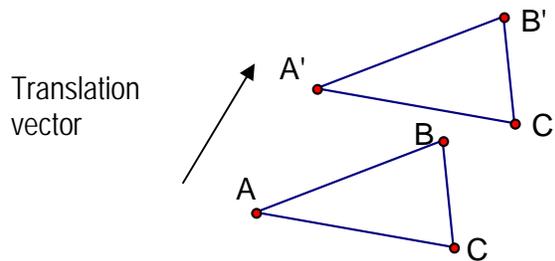
Figure 2

5. Using the pre-image below and the given translation vector in the direction. How could you find the image?
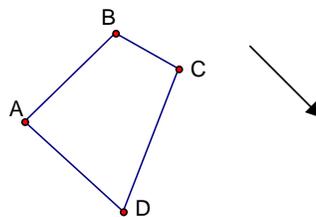
Figure 3

Part Two: Reflections

6. Go to the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) and choose 9-12 Geometry and then, choose Transformations—Reflections. Shown below is one possible screen shot.

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QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

a. Click on the shape and drag it. What happens?

b. Click on the black point and drag it. What happens?

c. Click on the line and drag it. What happens?

d. What do you think are some characteristics of reflections?

7. Now consider a different look at reflections Go to the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) and choose 9-12 Algebra and then, choose Grapher. Click on Function tab and type $f(x)=x^2+5$. Then, click on g(x) and type $g(x)=- (x^2+5)$. Then, click graph.

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

a. What do you notice about the graphs? About the equations?

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b. Now be sure the trace box is checked at the bottom right hand screen. Move the trace slider at the bottom of the screen. What do you notice about the values of f(x) and g(x) for any x-value?

c. How does this compare to the first example with reflections?

8. Using the image gallery that was created from Activity 1, choose an image that you believe has reflectional symmetry. Find the line of symmetry and place it on the picture. The can be done on paper or pasted into Sketchpad. Be specific about how you know that this is the line of symmetry.

9. For the preimage below, find the reflection about line *j*. Be sure to consider that the segment connecting a point and its image is perpendicular to line *j*.
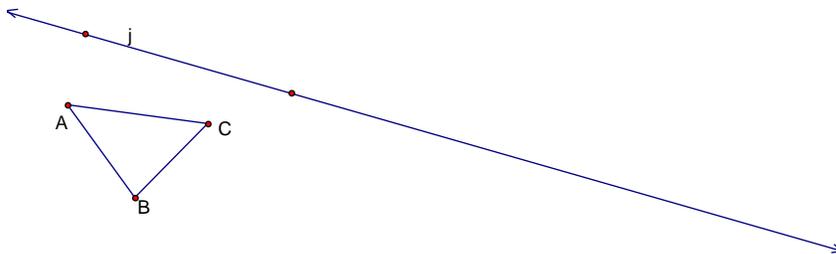
Figure 9

Summarize

Look back at your definition of translation and reflection Activity 1. Revise your definition based on the information from this activity. Use the following vocabulary in your definitions: isometry, vector, line of symmetry, and perpendicular bisector. Draw an example to be included with each of your definitions and include appropriate geometric markings.

Check Your Understanding

1. Given the preimage and image, draw in the translation vector.

2. Create your own figure and create a translation.

3. Reflect the figure over the line of symmetry.
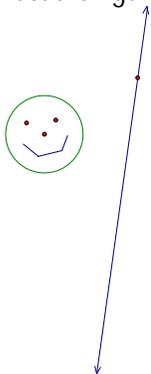
4. Create your own figure and create a reflection.

ACTIVITY 3

Activity 3 Overview: Rotations and Dilations

Iowa Core Curriculum Essential Skills/Concepts:	° Understands and applies transformations ° Uses the language of mathematics to express mathematical ideas precisely ° Uses representations to model and interpret physical, social, and mathematical phenomena ° Recognizes and uses connections among mathematical ideas and how they build on one another to produce a coherent whole ° Recognizes and applies mathematics in contexts outside of mathematics				
Big Ideas	° Creation of rotations require a center and angle of rotation ° Creation of dilations require a center and magnification factor ° Rotations and Dilations are identified in nature, graphic design, and art by their key features. ° Students use technology as a tool to make sense of transformations and to create transformations. ° There are transformations that preserve shape and/or distance.				
Characteristics of Instructional Core	x	Important core content that is evidence-based		Deep conceptual and procedural knowledge	x Reflective teaching in a collaborative setting
	x	Teaching for understanding		Rigorous curriculum	Development of global perspective
	x	Teaching through problem solving and inquiry	x	Relevant curriculum	x Incorporation of current technology
	x	Student-centered classrooms	x	Assessment for learning	
Cognitive Domain	x	Remembering	x	Applying	Evaluating
	x	Understanding		Analyzing	Creating
Connections to Students' Lives		Isolated within discipline		Connected to other disciplines	x Connected to student lives
	x	Connected within discipline		Has value beyond school purposes	
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume	
		Increasing access to print		Engaging students with texts	
		Involving students in discussion		Reading aloud in content areas	
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension	
		Writing to learn across content areas			
Class Time	1 class period(s) 50 total minutes				

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Materials & Set-up:
Ideally, all students would have a computer with an internet connection. If not, this can be done with one computer as a demonstration. Students will also need a protractor and a ruler. A digital camera for students would be ideal, but is not necessary. Also, Geometer's Sketchpad can be used during the activity.

Teaching Tips:
The purpose of this investigation is to introduce students to the different uses of rotations and dilations. It also intends to formalize the definition of rotation and dilation. Activity 2 is parallel to this activity, but focuses on translations and reflections.

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Teacher Tasks	Student Tasks
• Facilitate discussion of launch focusing on more specific mathematics included in a rotation. • Facilitate student learning through the rotation questions. • May want to have a mini-summary after the rotation questions. • Facilitate student learning through the dilation questions. • Facilitate the summary at the end of the task.	• During launch add suggestions of more specific mathematics needed for a rotation. • Engage in questions on student handout regarding rotations. Actively question and explore to understand the mathematics. • Take notes during a mini-summary. • Engage in questions on student handout regarding dilations. Actively question and explore to understand the mathematics. • Take notes and contribute to the summarize.

Differentiation Tasks

<u>Less-Than-Proficient</u>
Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction:
- vocabulary: isometry, center of rotation, directed angle, angle of rotation, center of dilation, scale factor
- skills/concepts: constructing a rotation and dilation by hand and using the computer software.

Supplemental instruction suggestions:
- Strategic partnerships for group work
- Create scaffolded notes for summary
- Have a Geometer's Sketchpad quick key stroke guide for students.

<u>Highly Proficient</u>
Supplemental instruction suggestions:
- Pretesting may suggest the need for compacting of some activities
- Have students work at an accelerated pace and provide questions about composition of two rotations and two dilations.

Intensive instructional suggestions for students who are highly proficient
- Pretesting may show a clear need for an alternative unit using college-level geometry materials.

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Formative Assessments:
- o Listen carefully to student responses to key questions in the student handout:
     What do you think are some characteristics of rotations?
     What do you think are some characteristics of dilations?
- o The summarize should be used to listen to student understanding and make instructional decisions on the next moves depending on the depth of understanding.

Summative Assessments:
- o The "Check for Understanding" can be used as a more summative assessment. Also Activity 8 will serve as a part of a summative assessment for the concepts in this unit.

Reflect:

The students seemed to enjoy the lesson and the different resources used throughout the activity. The connections between strands of mathematics are not as complete in this lesson and can be improved. This needed to be facilitated much more by the instructor than anticipated. By the end of the investigation students seemed to have a good understanding of rotation and dilation. Completing a rotation by hand was not as intuitive as I believed it would be after the opportunities presented in the lesson. Students especially enjoyed the symmetry artist website. It was difficult to have students move on as they wanted to "play" with the site.

Suggested Student Solutions

1. a) The preimage remains stationary while the image rotates. The angle changes.
    b) The images change location while maintaining the same angle. Students may have a variety of responses.
    c) Rotations have a directed angle and specific distance from the center of rotation.
2. a) Students should locate center more informally, although this is a possible place for more formal mathematics to occur.
    b) 60 degrees
    c) order 8
    d) Students should locate center more informally, although this is a possible place for more formal mathematics to occur.
    e) 45 degrees
3.
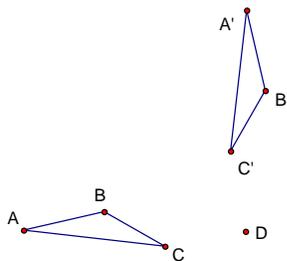
4.
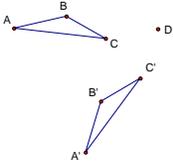
5. Here is a set of steps you can provide your students if you wish.

Step 1: To rotate point C, draw the segment connecting points C and D.
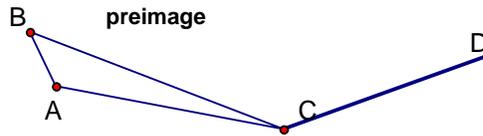
Step 2: Using your protractor, create a line that is 50 degrees in a counterclockwise direction from CD.
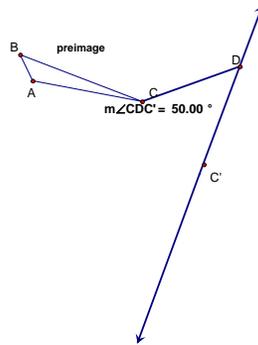
Step 3: Create C′ so that it is the same distance from D as C.

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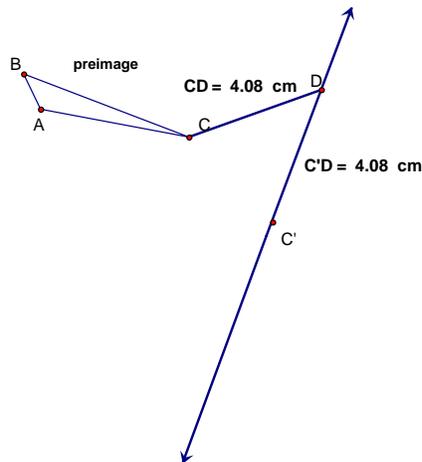
Step 4: Repeat these steps for point B as shown.

B

BD = 8.66 cm

D

m∠BDB' = 50.00 °

A

C

B'D = 8.66 cm

C'

B'
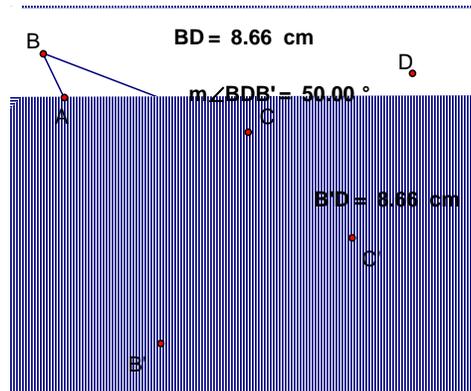
Step 5: Repeat these steps for point A. Then, connect your three image points to create your image triangle.

6. Answers will depend on student choices.
7. Students should locate two point sets of corresponding points and create the line to find the center of dilation. Then measure from the center of dilation to each point to find the dilation factor.
8. Answers will depend on student choices.
9. Angles are congruent
10. Lengths are doubled.

Summarize
Rotations are isometries determined by the center of rotation and a directed angle of rotation.
A **dilation** is a size tranformation determined by a center and a factor.

Activity 3
Student Page/Handout

Rotations and Dilations

There are several different types of symmetries and transformations. In translations and reflections, characteristics such as angles, side lengths, distances, shapes, and sizes are maintained. However, rotations and dilations may produce a different type of symmetry. There are also transformations that do not create symmetry but similarity.

Launch

One student chose the artwork below and had grouped it under rotational symmetry. This artwork has rotational symmetry of 120 degrees as shown below. What other rotational symmetry exists for this artwork?
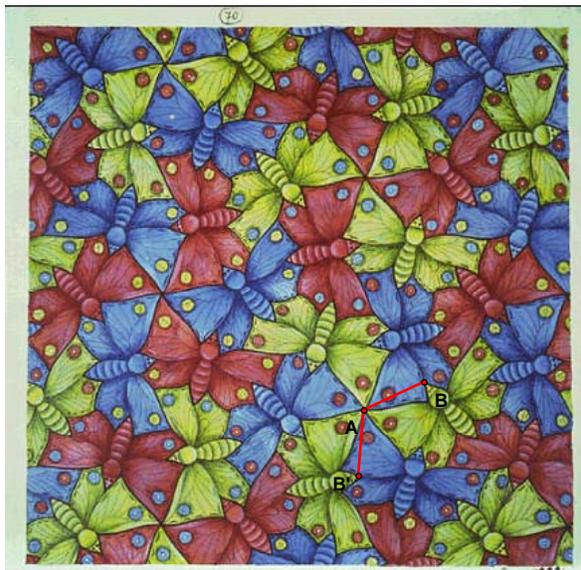
$m\angle BAB' = 120.00°$

Figure 1

Think about how you could more precisely classify some of the examples from each group from the launch in Activity 1. What would you need to identify?

Exploration

Part One: Rotations
1. Go to the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) and choose 9-12 Geometry and then, choose Transformations—Rotations.

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

*Taken from http://nlvm.usu.edu/en/nav/category_g_4_t_3.html

a)   Click on the shape and drag it. What happens?

b)   Click on the point and drag it. What happens?

c)   What do you think are some characteristics of rotations?

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2. Go to http://www.mathsisfun.com/geometry/symmetry-rotational.html and read about rotational symmetry.

image 1

preimage
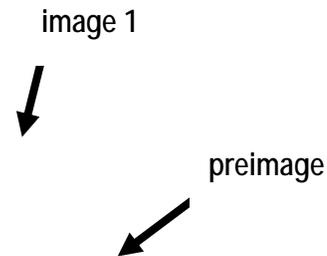
QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

a) The picture in the top right corner has order 6 symmetry. Locate where you believe the center of rotation is? Explain why you chose this point.

b) How many degrees do you think the preimage would have to be rotated to match up to the image? Why?

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image

preimage
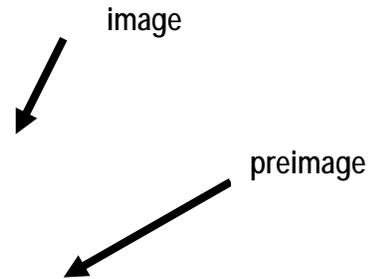
QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

c) What order symmetry does the picture on the top left corner have?

d) Locate where you believe the center of rotation is? Explain why you chose this point.

e) How many degrees do you think the pre-image would have to be rotated to match up to the image? Why?

3. On your paper *sketch* the image triangle for a clockwise (negative) rotation of 90 degrees about the point D.

B

A

C

• D
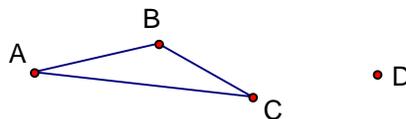
Figure 3

4. On your paper *sketch* the image triangle for a counterclockwise (positive) rotation of 60 degree about the point D.
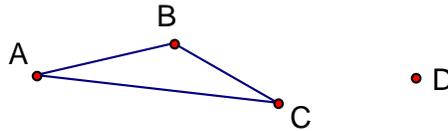
Figure 4

5. Using a ruler and protractor draw a counterclockwise rotation of 50 degrees as accurately as possible.
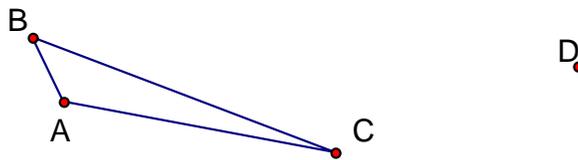
Figure 5

6. Using the image gallery that was created from the Check Your Understanding in Activity 1, choose an image to find the center and angle of rotation. These can be done using sketchpad or paper and pencil. If you use a digital image, copy and paste it into Sketchpad. On your image, state the center and angle of rotation. Explain how you found each.

Part Two: Dilations

7. What is the dilation factor between the smallest person in the myspace logo and the largest person? Explain how you found it. Can you locate the center of the dilation? How?

8. Create a shape on Sketchpad or on your paper. Create a dilation with a factor of 2.

9. What is true about the angles of the preimage and the image figures?

10. What is true about the lengths of the line segments?

Summarize

Look back at your definition of rotation and dilation from Activity 1. Revise your definitions based on the information from this activity. Use the following vocabulary in your definitions: isometry, center, magnification factor, angle of rotation, and direction of rotation. Draw examples to be included with your definitions and include appropriate geometric markings.

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Check Your Understanding

1. Find the center of rotation.

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

2. Rotate the figure about A. Choose an angle of rotation. Verify that your rotation is accurate.

A

3. Create a dilation of a figure with a magnification factor of 3. Verify that your dilation is accurate.

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ACTIVITY 4

Activity 4 Overview: Transformations on the Coordinate Plane

Iowa Core Curriculum Essential Skills/Concepts:	° Understands and applies transformations ° Understands and applies coordinates				
Big Ideas	° Students should understand and use transformations that preserve distance (called isometries or rigid motions, such as reflections, rotations, translations) ° Students should understand and use transformations that preserve shape (such as size transformations, dilations, or similarity transformations) ° Students should understand and use transformations that change distance and shape (e.g., shears) ° Students should be able to identify, create, describe, and justify transformations using multiple representations. They should be able to find and describe an image under a given transformation and/or composition of transformations. Students should also be able to identify the transformations that produced a given image. Transformations should be represented algebraically (using coordinate rules, matrices, vectors, equations), and those representations should be used to analyze and reason about transformations.				
Characteristics of Instructional Core	x	Important core content that is evidence-based	x	Deep conceptual and procedural knowledge	x	Reflective teaching in a collaborative setting
	x	Teaching for understanding	x	Rigorous curriculum		Development of global perspective
	x	Teaching through problem solving and inquiry		Relevant curriculum	x	Incorporation of current technology
	x	Student-centered classrooms	x	Assessment for learning		
Cognitive Domain	x	Remembering	x	Applying		Evaluating
	x	Understanding		Analyzing		Creating
Connections to Students' Lives		Isolated within discipline		Connected to other disciplines		Connected to student lives
	x	Connected within discipline		Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		
		Increasing access to print		Engaging students with texts		
		Involving students in discussion		Reading aloud in content areas		
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension		
		Writing to learn across content areas				
Class Time	1 class period(s) 50 total minutes				

Materials & Set-up:
Graphing calculators for each student and a projection device
Copies of student handouts

Teaching Tips:
It is helpful for the students to be fluent in the use of the graphing calculator. If the class is not, the teacher may want to provide extra calculator instructions or teach a student ahead of time to help out.

Teacher Tasks	Student Tasks
• On the overhead, guide students through drawing the letter F and have students generate two **lists** and use **stat-plot** to display. Use **zoom-stat**, then **zoom-square**. (Launch) • Facilitate discussion about questions (a) and (b) • (Launch) • For #1-? Teacher guided through questioning students about how they think the transformations can occur. A student uses the overhead calculator while other students generate shapes on theirs.	• Creating the letter F on their calculator using lists. • Moving the letter F about the Cartesian Plane numerically by using coordinates.
Differentiation Tasks	

Less-Than-Proficient
Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction:
  • vocabulary: translate, reflect, rotate, dilate, shear, stretch, shrink
  • skills/concepts:
Supplemental instruction suggestions:
  • This task can be paced for all students to be successful. Letting students set a reasonable pace may be important.
  • Strategic partnerships

Highly Proficient
Supplemental instruction suggestions:
  • Giving students freedom to accelerate through the activity will help the highly proficient. Working as a whole class may be frustrating.
  • Strategic partnerships

Assessment:

Formative Assessments:

o As the students progress through the activity, ask about the thought process they are going through to predict the calculator commands and the actual moves. This activity is frequently self-checking through the use of the calculator so students will be provided feedback on their accuracy.

Summative Assessment:
o The Check for Understanding is quite complete for this activity and can be used in either a formative or summative way.

Reflect:

As long as the calculator works well for students, the students seem to enjoy the activity. Students demonstrated proficiency in the movement of the letter F using the appropriate transformations. Although most of the learning was student driven, it is helpful to do a thorough summary for all students to synthesize the information.

Suggested solutions to Student Handout:

Launch:
Students will need the following calculator screens to create the letter F.
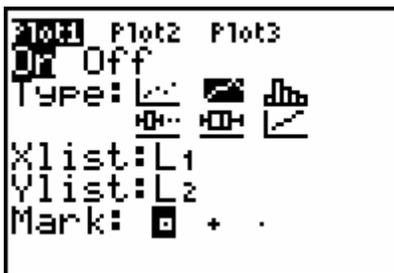
a) They would need to add 5 units to each y value in the list.
b) Let students explore possibilities. They do not have to have accurate solutions at this point. List the students' predictions to check for accuracy throughout the lesson.

Explore:
1. The X-list is changed to –X and stored in a list which is named REFLY.

2.

3.
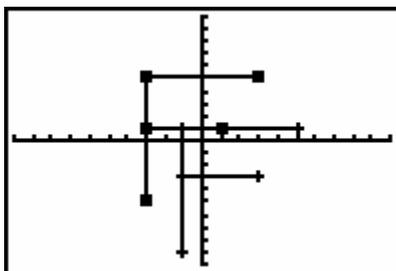
4.

```
2*L₂→LSTREV
(10 10 2 2 2 -1…
■
```

```
STAT PLOTS
1:Plot1…Off
   ⌐ L1    L2    .
2:Plot2…On
   ⌐ L1    STREV ▫
3:Plot3…Off
   ⌐ L1    UPDN  +
4↓PlotsOff
```
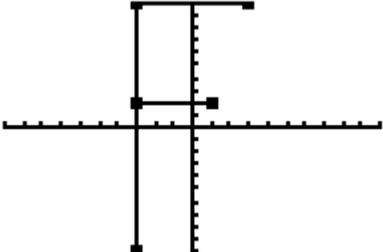
5. List 1 should be multiplied by a factor greater than 1.

6. List 1 should be multiplied by a factor between 0 and 1.

Summarize

Reflection over the x-axis	(x,y) ———▶(*x,-y*)
Reflection over the y-axis	(x,y) ———▶(*-x,y*)
Horizontal Translation to the Right a units	(x,y) ———▶(x+a_,_y_) a>0
Vertical Translation Down a units	(x,y) ———▶(x,_y+a) a<0
Vertical stretch by a factor of *a*, *a>1*	(x,y) ———▶(*x,ay*)
Horizontal Shrink by a factor of a	(x,y) ———▶(ax , y) 0<a<1

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Check For Understanding

1. Which list represents the x-coordinate? Which list represents the y-coordinate?

   X-list = L_1 Y-list = L_2

2. *Reflect across the y-axis*
   X-list = $-L_1$ Y-list = L_2

3. *Reflect over the x-axis and over the y-axis*

   X-list = - L_1 Y-list = $-L_2$

4. *Reflect over the x-axis*

   X-list = L_1 Y-list = $-L_2$

5. X-list = L_1 Y-list = L_2

6. *Move 6 units to the left*
   X-list = $L_1 - 6$ Y-list = L_2

7. *Move Down 7 units*

   X-list = L_1 Y-list = $L_2 - 7$

8. *Translate Left 6 and Down 7*
   X-list = $L_1 - 6$ Y-list = $L_2 - 7$

9. *Reflect across the x-axis and then translate up 7 units*

   X-list = L_1 Y-list = - $L_2 + 7$

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10. X-list = $-L_1$ Y-list = $-L_2 + 7$

11. X-list = L_1 Y-list = $L_2 - 7$

12. X-list = $-L_1$ Y-list = $L_2 - 7$

Stopper

Translate List 1 seven units to the left. Reflect List 5 and List 6 in the x-axis and then move up 9 units.

Activity 4 Student Page/Handout

Transformations on the Coordinate Plane

Launch

Draw the letter F with the given coordinates. Use **zoom-stat**, then **zoom-square** when graphing.
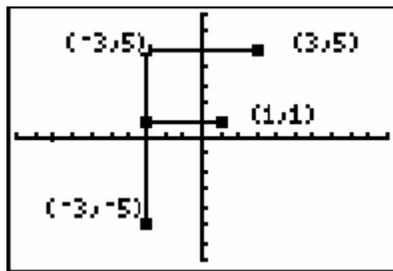
c)   If you want to move the letter F up to the x-axis, what do you think you would do to each coordinate?

d)   How else might you transform this shape? What would you have to do with the coordinates to make the transformation appear?

Exploration

1.   If the letter F is reflected across the y-axis, what needs to be done to the lists? Store your solution in REFLY.

2.   With a partner, create a list named REFLX and reflect the original letter F across the X-axis.

3.   Now move the original letter F right 2 and down 4. Create two new lists named RTLFT and UPDN. Change the window to **Zoom-Standard**. What transformation is this?

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4. Create a list named STRETV and stretch the letter F in the vertical direction by a factor of 2.

5. What would you do to stretch the letter F in the horizontal direction?

6. If you want to shrink the letter F, what would you use as a factor?

Summarize

For each transformation complete the coordinate mapping or identify the transformation. Look back at your calculator work to help you.

Reflection over the x-axis	$(x,y) \longrightarrow (\_\_\_\_,\_\_\_\_)$
Reflection over the y-axis	$(x,y) \longrightarrow (\_\_\_\_,\_\_\_\_)$
	$(x,y) \longrightarrow (x+a\_\_,\_y\_)$ a>0
	$(x,y) \longrightarrow (x,\_y+a)$ a<0
Vertical stretch by a factor of *a*, *a>1*	$(x,y) \longrightarrow (\_\_\_\_,\_\_\_\_)$
	$(x,y) \longrightarrow (ax, y)$, 0<a<1

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Check For Understanding

Enter the following lists. Close the figure by putting another zero at the end of L1 and a 7 at the end of L2.

Calculator List	Original Figure	Transformed figures

1. Which list represents the x-coordinate? Which list represents the y-coordinate?

    X-list _____   Y-list _____

2. How would you move the figure from quadrant one to quadrant two? Indicate what you would do to list 1 and/or list 2. Check it on your calculator.

    X-list _____   Y-list _____

3. How would you move the figure from quadrant one to quadrant three? Indicate what you would do to list 1 and/or list 2. Check it on your calculator.

    X-list _____   Y-list _____

4. How would you move the figure from quadrant one to quadrant four? Indicate what you would do to list 1 and/or list 2. Check it on your calculator.

    X-list _____   Y-list _____

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Use the figures below for #5-8.

Calculator List	Original Figure	Transformed figures

5. Which list represents the x-coordinate? Which list represents the y-coordinate?

X-list _____   Y-list _____

6.   How would you move the figure from quadrant one to quadrant two? Indicate what you would do to list 1 and/or list 2. Check it on your calculator.

X-list _____   Y-list _____

7.   How would you move the figure from quadrant one to quadrant four? Indicate what you would do to list 1 and/or list 2. Check it on your calculator.

X-list _____   Y-list _____

8.   How would you move the figure from quadrant one to quadrant three? Indicate what you would do to list 1 and/or list 2. Check it on your calculator.

X-list _____   Y-list _____

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Calculator List	New Figure for Quadrant 1	Transformed figures

9. What would you do to list 1 and list 2 to obtain the figure in Quadrant 1?

    X-list _____   Y-list _____

10. What would you do to list 1 and list 2 to obtain the figure in Quadrant 2? Check it on your calculator.

    X-list _____   Y-list _____

11. What would you do to list 1 and list 2 to obtain the figure in Quadrant 4? Check it on your calculator.

    X-list _____   Y-list _____

12. What would you do to list 1 and list 2 to obtain the figure in Quadrant 3? Check it on your calculator.

    X-list _____   Y-list _____

Enter L1, L2, L5, and L6 and then connect the pieces as shown. Braces are found above the parenthesis. Remember to press ENTER.

Operate on the jar and stopper until the 2 pieces are assembled. You may assemble in additional quadrants. Explain each step you did to complete the assembly.

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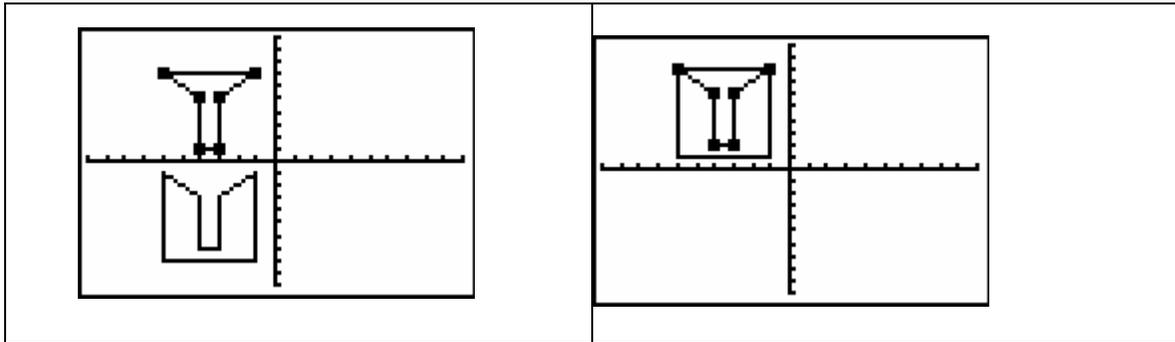
ACTIVITY 5

Activity 5 Overview: Transformations with Matrices

Iowa Core Curriculum Essential Skills/Concepts:	° Understands and applies transformations ° Understands and applies coordinates				
Big Ideas	° Students should understand and use transformations that preserve distance (called isometries or rigid motions, such as reflections, rotations, translations) ° Students should understand and use transformations that preserve shape (such as size transformations, dilations, or similarity transformations) ° Students should understand and use transformations that change distance and shape (e.g., shears) ° Students should be able to identify, create, describe, and justify transformations using multiple representations. They should be able to find and describe an image under a given transformation and/or composition of transformations. Students should also be able to identify the transformations that produced a given image. Transformations should be represented algebraically (using coordinate rules, matrices, vectors, equations), and those representations should be used to analyze and reason about transformations.				
Characteristics of Instructional Core	x	Important core content that is evidence-based	x	Deep conceptual and procedural knowledge	x	Reflective teaching in a collaborative setting
	x	Teaching for understanding		Rigorous curriculum		Development of global perspective
	x	Teaching through problem solving and inquiry		Relevant curriculum	x	Incorporation of current technology
	x	Student-centered classrooms	x	Assessment for learning		
Cognitive Domain	x	Remembering	x	Applying		Evaluating
	x	Understanding	x	Analyzing		Creating
Connections to Students' Lives		Isolated within discipline		Connected to other disciplines		Connected to student lives
	x	Connected within discipline		Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		
		Increasing access to print		Engaging students with texts		
		Involving students in discussion		Reading aloud in content areas		
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension		
		Writing to learn across content areas				
Class Time	2 class period(s) 100 total minutes				

Materials & Set-up:
Students will need graphing calculators. A projection device will be helpful for this activity. Students will also need the student handouts

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Teaching Tips:
This activity parallels activity four but highlights matrices as tool for representing the transformations. Graphing calculators are critical for success. You may want to introduce Activity 8 after this activity. It will give students sometime to work on their projects. This lesson functions well with partnerships.

Teacher Tasks	Student Tasks
• On the overhead guide students through drawing the letter F and have students generate two **lists** and use **stat-plot** to display. Use **zoom-stat**, then **zoom-square**. • Teacher facilitates through questioning students about how they think the transformations can occur. A student uses the overhead calculator while other students generate shapes on theirs. • Facilitates a summary for the student.	• Create the letter F on their calculator using matrices and lists. • Hypothesize about what matrices will work for different transformations. • Self-check and correct using the calculator. • Take notes and contribute to the summary

Differentiation Tasks

Less-Than-Proficient
Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction:
- vocabulary: translate, reflect, rotate, dilate, shear, stretch, shrink
- skills/concepts:

Supplemental instruction suggestions:
This task can be paced for all students to be successful. Letting students set a reasonable pace may be important.

Highly Proficient
Supplemental instruction suggestions:
Giving students freedom to accelerate through the activity will help the highly proficient. Working as a whole class may be frustrating.

Intensive instructional suggestions for students who are highly proficient
- The matrix representation for a rotation is not given here, but may be ideal for the highly proficient. ($\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$)

```
MATRIX[B] 2 ×2
[ .70711  -.7071    ]
[ .70711  .70711    ]




2,1=sin(45)█
```

Let θ = 45 degrees.

Assessment:

Formative Assessments:
- o As the students progress through the activity, ask about the thought process they are going through to predict the calculator commands and the actual moves. This activity is frequently self-checking through the use of the calculator so students will be provided feedback on their accuracy.

Summative Assessment
- o The Check for Understanding is quite complete for this activity and can be used in either a formative or summative way.

Reflect:

As long as the calculator works well for students, the students seem to enjoy the activity. Students demonstrated proficiency in creating matrices for the appropriate transformations. Although most of the learning was student driven, it is helpful to do a thorough summary for all students to synthesize the information.

Solutions to Student Handout.

1. a) Ask "Which coordinate is changed?"   b) Add Matrix A to Matrix B

```
MATRIX[B] 6 ×2
[ 4        0        ]
[ 4        0        ]
[ 4        0        ]
[ 4        0        ]
[ 4        0        ]
[ 4        0        ]

6,2=0
```

```
[A]+[B]→[C]
        [[7  5 ]
         [1  5 ]
         [1  1 ]
         [5  1 ]
         [1  1 ]
         [1  -5]]
```

2.

3.

Reflection

4. a) The x-coordinate is the opposite and the y-coordinate stays the same. Thus, the x-coordinate needs to be multiplied by -1.

b)

c)

5. a) The y-coordinate is multiplied by negative one. The x-coordinate stays the same.

b)

6 . a) The x-coordinate will change by a factor of 3.
   b) The transformation matrix [B] and sequence of operations is as follows.
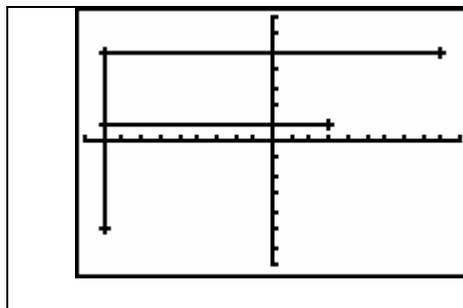
7. Multiply the y-coordinate by 5. The matrix is given below.

x-min = -30, x-max=30
y-min = -30, y-max = 30

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8. a) This is a vertical shrink by a factor of 0.6.
    b) This is a horizontal shrink by a factor of 0.5.
    c) This is a dilation by a factor of 2.
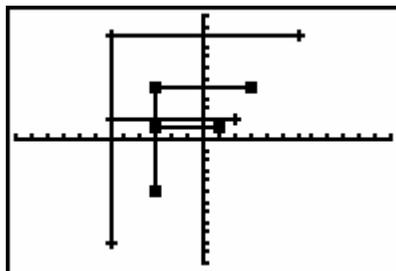
SUMMARIZE

Transformation	Matrix Representation
Reflection over the y-axis	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
Reflection over the x-axis	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Translation *a* units horizontally and *b* units vertically.	$\begin{bmatrix} a & b \\ a & b \\ \dots & \dots \end{bmatrix}$
Horizontal Shrink by a factor of *a*	$\begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}$, $0<a<1$
Vertical stretch by a factor of *a*, if *a>1*.	$\begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$
Horizontal Stretch if a>1 or shrink if 0<a<1. *Vertical Stretch if b>1 or shrink if 0<b<1*	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

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Activity 5 Transformations with Matrices
Student Page/Handout

Launch

Next matrices will be used to graph translated, reflected, rotated, dilated, and sheared diagrams. A matrix is used to organize data.
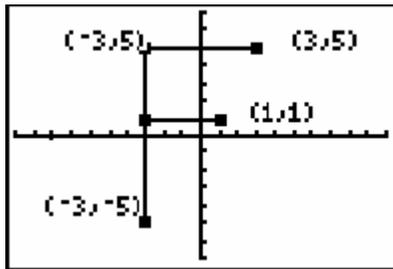
a) How can you represent the letter F using a matrix?

b) Think about how you might use matrices to translate the letter F to the right 2 and down 4. Share your ideas with the class.

Explore

First take the lists and put them into a matrix. To do this Press matrix, math, 9, 2nd, L1, comma, 2nd, L2, comma, matrix,[A],), enter. The following screen shots may be helpful.
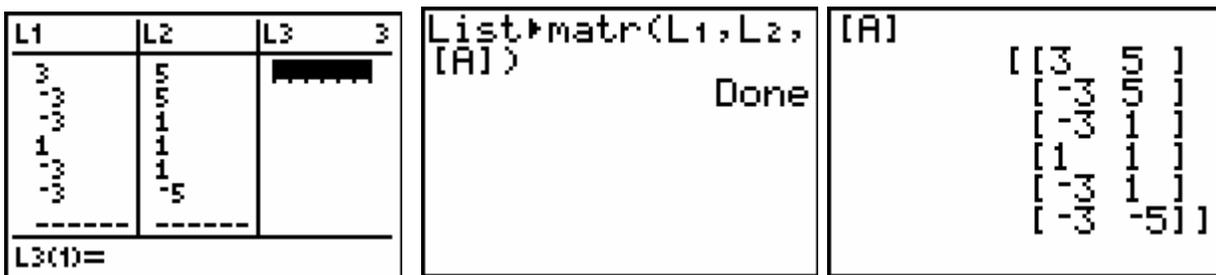
Translations

1. a) Create a 6x2 **Matrix B** that will move the letter F right 4 and leave the y-coordinate the same. To enter this matrix in your calculator press matrix, edit, 2. Change the dimensions and enter your values.

b)   What operation on Matrix A and Matrix B will translate the letter F?

c)   In order to get a new matrix that can be plotted we need to store the result of Matrix A and Matrix B in Matrix C as shown below.

```
[A]+[B]→[C]
          [[7  5 ]
           [1  5 ]
           [1  1 ]
           [5  1 ]
           [1  1 ]
           [1  -5]]
```

d)   Now change Matrix [C] to lists so stat-plot can be used to graph the translated F. Use zoom 6, zoom square for the window. The screen shots below may be helpful.

```
Matr►list([C],ʟR
TLFT,L₂)
            Done
```

```
STAT PLOTS
1:Plot1…On
   ⌐ L1    L2    ▫
2:Plot2…On
   ⌐ RTLFT L2   +
3:Plot3…Off
   ⌐ RTLFT L2   ▫
4↓PlotsOff
```
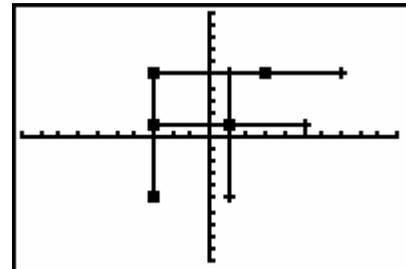
2.   Create a Matrix B to translate the original F down 3.   Operate on Matrix A and Matrix B and store the result in Matrix C. Plot your image F using the screen shot below to help you. Remember to zoom 9 and zoom square.

```
Matr►list([C],L₁
, ʟUPDN)
            Done
■
```

```
STAT PLOTS
1:Plot1…On
   ⌐ L1    L2    ▫
2:Plot2…Off
   ⌐ RTLFT L2   +
3:Plot3…On
   ⌐ L1    UPDN  +
4↓PlotsOff
```

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3. Create a matrix that would move the original F right 4 and down 6. The screen shots may help you plot your new F.

REFLECTION

Reflection across both the x-and y-axis will involve multiplication of matrices.
First, matrix [A] must be transposed. Press matrix, [A], enter, matrix, math, *T*.
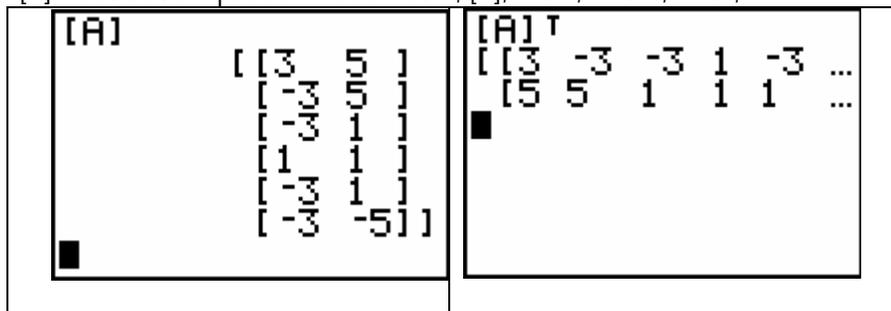
4. Matrices can be used to represent reflections as well. To simplify this, let's reflect just point A. We know that A (3,5) is reflected about the y-axis to A' (-3, 5). We need to find the 2x2 matrix that when we multiply it by the matrix representing the point (3,5), we get the matrix representing (-3,5).

We know: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

a) Which coordinate will change if F is reflected across the y-axis? What stays the same? How will it change?

b) Using your answers in (a) and what you already know about know about matrices. For a reflection about the y-axis we want

$\begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$ What values go in each blank?

The matrix above should represent a reflection in the y-axis for any point (x, y). Enter the above matrix in Matrix [B].

c) Now graph the reflected F using stat-plot. Matrix [C] must be placed into new lists, named NewX and NewY.

```
NEWX    NEWY    ▓     5 |Matr▸list([C]ᵀ, L |Plot1  ▓▓▓  Plot3
------  ------  ------ |NEWX, LNEWY)       |░░ Off
                      |          Done     |Type:▱ ▨ ⬛
                      |■                  |     ▱  ▱  ▱
                      |                   |Xlist:NEWX
                      |                   |Ylist:NEWY
L3 =                  |                   |Mark:  ▫ ■ ·
```

5. Find the matrix representation for a reflection in the x-axis for any point (x,y). First ask yourself the following questions.

   a. What coordinate will change? How will it change? What will stay the same?

   b) Find the matrix representation for a reflection in the x-axis for any point (x,y) by filling in the missing blanks by using your answers in (a) and what we know about matrices.

$$\begin{bmatrix} \rule{1cm}{0.4pt} & \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt} & \rule{1cm}{0.4pt} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} x \\ -y \end{bmatrix}$$ **What values go in each blank?**

The matrix above should represent a reflection in the x-axis for any point (x, y). Enter the above matrix in Matrix [B].

```
[B]*[A]ᵀ→[C]        |Matr▸list([C]ᵀ, L
[[3   -3 -3 1  -… |NEWX, LNEWY)
 [-5 -5 -1 -1 -… |          Done
                  |■
```

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Stretch

6. a) To expand or stretch the original letter F horizontally by a factor of 3, what coordinate will change? How will it change?

   b) Find the matrix representation for a horizontal stretch by a factor of 3. Verify you are correct by using your calculator.

7. What would you do to expand the letter F vertically by a factor of five using matrices?

8. Find the matrix representations for the following transformations.

a)   Preimage (original figure)   Image (new figure)

Explain your process:

b)   Preimage (original figure)   Image (new figure)

Explain your process.

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c)   Preimage (original figure)   Image (new figure)
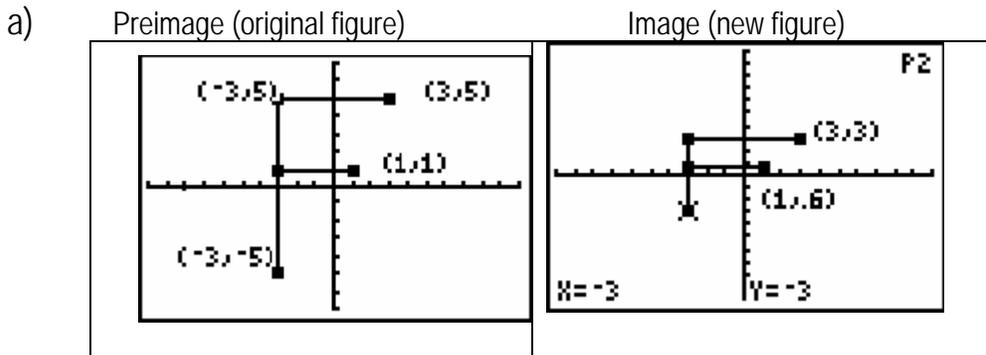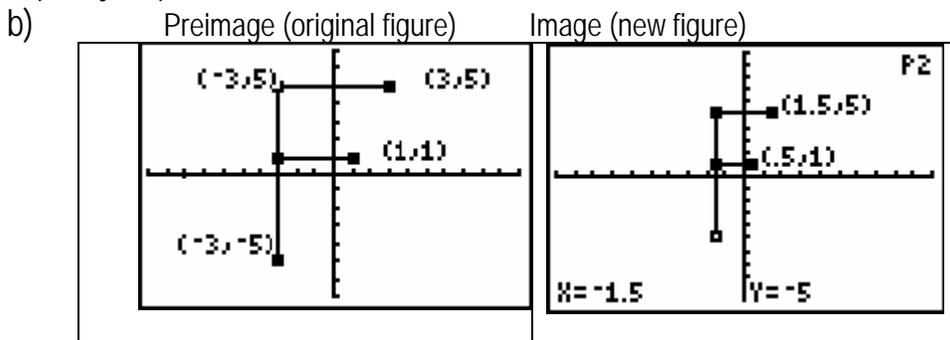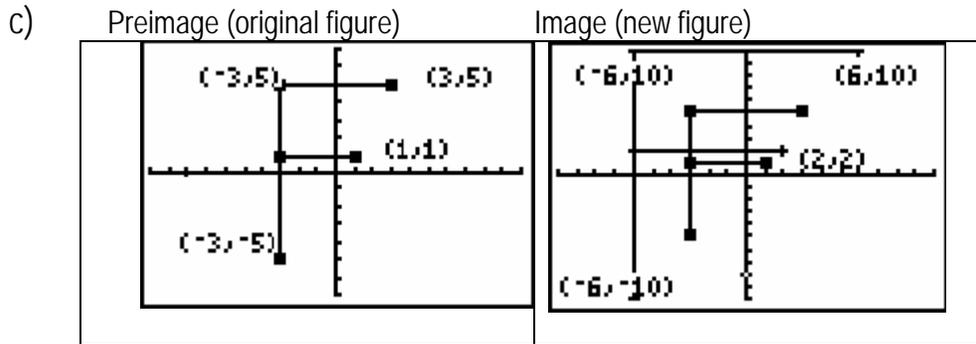
Explain your process.

Summarize

For each transformation complete the matrix representation for the transformation or identify the transformation. Look back at your calculator work to help you.

Transformation	Matrix Representation
	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
Reflection over the x-axis	
Translation *a* units horizontally and *b* units vertically.	
	$\begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}$, 0<*a*<1
Vertical stretch by a factor of *a*, *a>1*	
	$\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

Check For Understanding

Create your own figure on the calculator. Complete at least three transformations using matrices. Explain or show what you did. Here are a couple figures if you want to start with them instead of making your own figure.

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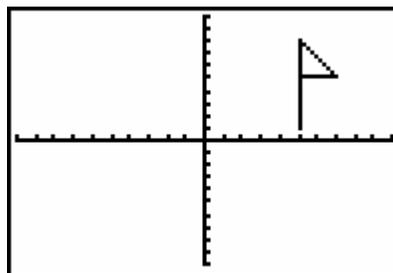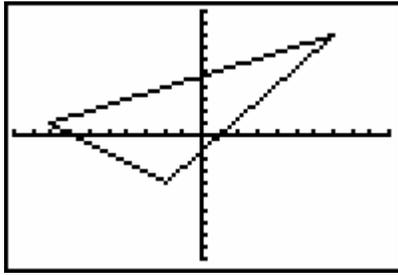
ACTIVITY 6

Activity 6 Overview: *Transformations of Functions Computer Version OR Calculator Version (Both are attached after the activity overview)*

Iowa Core Curriculum Essential Skills/Concepts:	° Understands and applies functions ° Understands and applies transformations				
Big Ideas	° Analysis of these functions should include: zeros, maximum and minimum, domain and range, global and local behavior, intercepts, rates of change, and asymptotes. ° Student should understand and use transformations that preserve distance (called isometries or rigid motions, such as reflections, rotations, translations) ° Students should understand and use transformations that preserve shape (such as size transformations, dilations, or similarity transformations) ° Student should understand and use transformations that change distance and shape (e.g., shears) ° Students should be able to identify, create, describe, and justify transformations using multiple representations. They should be able to find and describe an image under a given transformation and/or composition of transformations. Students should also be able to identify the transformations that produced a given image. Transformations should be represented algebraically (using coordinate rules, matrices, vectors, equations), and those representations should be used to analyze and reason about transformations.				
Characteristics of Instructional Core	X	Important core content that is evidence-based	X	Deep conceptual and procedural knowledge	X	Reflective teaching in a collaborative setting
	X	Teaching for understanding	X	Rigorous curriculum		Development of global perspective
	X	Teaching through problem solving and inquiry		Relevant curriculum	X	Incorporation of current technology
	X	Student-centered classrooms	X	Assessment for learning		
Cognitive Domain	X	Remembering	X	Applying		Evaluating
	X	Understanding	X	Analyzing		Creating
Connections to Students' Lives		Isolated within discipline		Connected to other disciplines		Connected to student lives
	X	Connected within discipline		Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		

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	Increasing access to print	Engaging students with texts
	Involving students in discussion	Reading aloud in content areas
	Increasing reading fluency	Explicitly instructing in vocabulary and comprehension
	Writing to learn across content areas	
Class Time	2 class period(s) approximately 100 total minutes	

Materials & Set-up:
Graphing calculators or a computer with CPMP-Tools can be used to complete this activity. Students will each need a copy of the handout
Teaching Tips:
A short overall summary.

Teacher Tasks	Student Tasks
• Teacher facilitates through questioning students about how they think the transformations can occur. • Facilitates the summary at the end of the task.	• Graph parent functions and follow through with investigations. • Actively thinks about the connections between transformations and functions • Creates a note sheet for summarizing.
Differentiation Tasks	

Less-Than-Proficient
Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction:
- vocabulary: function
- skills/concepts: Sketching a graph by associating it with a parent function and transformation or vice versa

Supplemental instruction suggestions:
- May want to create tech tips for some students.
- May want to work in a small group through a couple examples before working independently

Highly Proficient
Supplemental instruction suggestions:
- Can change parent functions to be more complex
- Extend the work with more compositions
- Give unknown functions and ask transformation questions about the unknown functions

Assessment:

Formative Assessments:
- o The summarize in this activity serves well to understand how students are understanding the concepts. In addition, listening and questioning students at the end of each "type" or transformation is helpful

Summative Assessments
- o The "Check for Understanding" can be used as a summative assessment. Activity 8 will not serve as a summative assessment for this activity.

Reflect:

This is a reflection about the computer software version of this lesson. It went smoothly with only some confusion of the difference between "y=" notation and function notation. Students might need a quick tutorial to use the software, but then it was used easily by most students.

Solutions to Student Handout.

1. a) Up 4
   b) Up 7
   c) Up 2
   d) Down 2
   e) Down 8
   f) Down 4
   g) Translation Up if k>0 and Translation Down if k<0.

2. a) Left 4
   b) Left 7
   c) Left 2
   d) Right 2
   e) Right 8
   f Right 4
   g) If $f(x)=(x-h)^2$, translate right if h>0 and translate left if h<0.

3. a) Translate right 4 and up 5.
   b) Translate right 4 and down 5.
   c) Translate left 4 and up 5.
   d) Translate left 4 and down 5.
   e) Translate right h if h>0 and left h if h<0, up k units if k>0 and down k units if k<0.

4. Translate right h if h>0 and left h if h<0, up k units if k>0 and down k units if k<0.

5. a) Reflect over x-axis
   b) Reflect over x-axis then translate up 7
   c) Reflect over x-axis then translate down 2
   d) Reflect over x-axis then translate left 7
   e) Reflect over x-axis then translate right 2
   f) Reflect over x-axis and translation (see 2g)
   g) Reflections over x-axis and translations left or right and up or down.
   h) Reflect over x-axis

6. a) Vertical Stretch by a factor of 2
   b) Vertical Stretch by a factor of 4
   c)Vertical Stretch by a factor of 6
   d) Vertical Shrink by a factor of ½
   e) Vertical Shrink by a factor of 1/3
   f) Vertical Shrink by a factor of 1/5
   g) Vertical Stretch if a>1 and a vertical shrink if 0<a<1. Reflection about x-axis if a=-1

7. Same as Problem # 6

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8.   *a) Horizontal Shrink by a factor of 2*
        b) Horizontal Shrink by a factor of 4
        c) Horizontal Shrink by a factor of 6
        d) Horizontal Stretch by a factor of 2
        e) Horizontal Stretch by a factor of 3
        f) Horizontal Stretch by a factor of 5
        g) Vertical stretch if a>1 and vertical shrink if 0<a<1 and reflection over the x-axis if a = -1
        h) Horizontal stretch if o<b<1 and a horizontal shrink if b>1 and reflection over the y-axis if b = -1.

Check for Understanding

Sketch the following graphs without the use of a calculator.
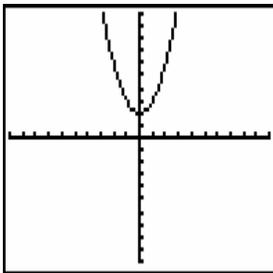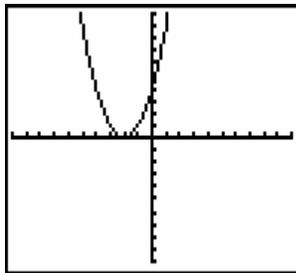
a) $f(x) = x^2 + 2$ b) $f(x) = (x+2)^2$ c) $f(x) = -|x-5|$

d). $f(x) = |x+3|$ e). $f(x) = -x^2 + 2$ f). $f(x) = -|x-3| + 6$

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         Month XX, XXXX
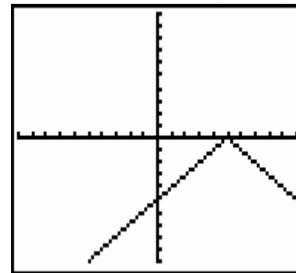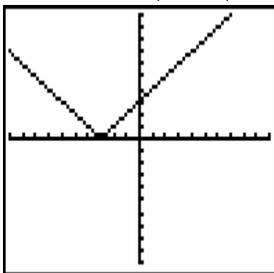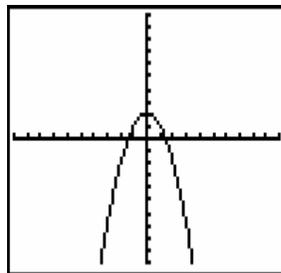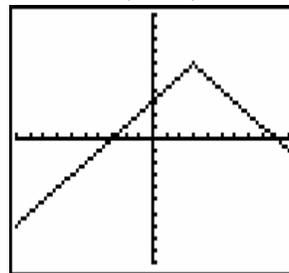
Given the following graphs, write the appropriate y = entries.

A) $f(x) = |x| - 2$

B) $f(x) = |x + 4|$
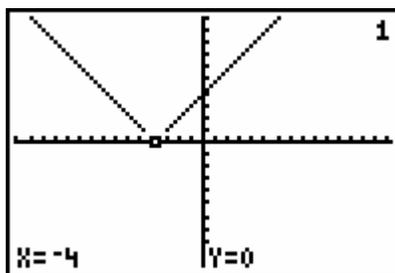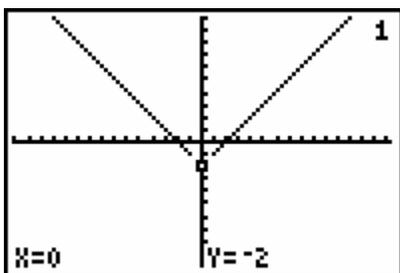
C) $f(x) = (x - 5)^2$

D) $x^2 + 6$
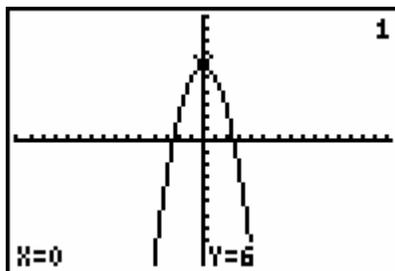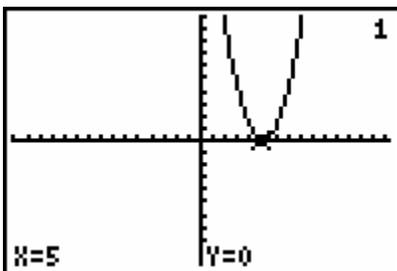
E) $f(x) = |x + 4| - 6$

F) $f(x) = -(x - 5)^2 + 4$
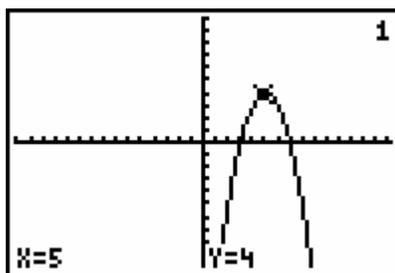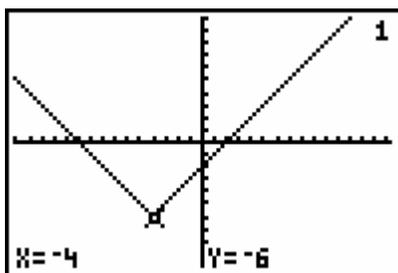
Your instructor will give you a parent function. Complete the following chart for your assigned function.

If f(x)=x

Function	Describe Translation	***SKETCH IT! ***
$y = f(x)$		
$y = f(x+3)$	Translate Left 3	
$y = f(x-3)$	Translate Right 3	
$y = f(x)-5$	Translate Down 5	
$y = f(x)+5$	Translate Up 5	

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Let $f(x) = x^2$
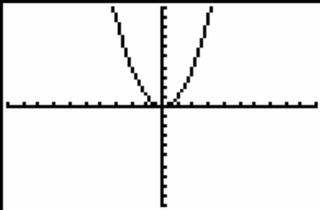
Function	Describe Translation	***SKETCH IT!***
$y = f(x)$		
$y = -f(x)$	Reflect over the x axis	
$y = f(-x)$	Reflect over the y-axis	
$y = 2f(x)$	Vertical Stretch by A factor of 2 (Thick line)	
$y = \dfrac{f(x)}{2}$	Vertical Shrink by A factor of 2 (Thick line)	
$y = f(3x)$	Horizontal Shrink	
$y = f(\dfrac{x}{3})$	Horizontal Stretch	
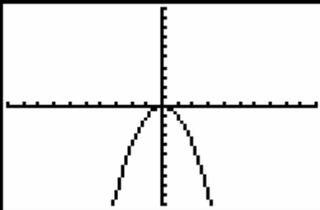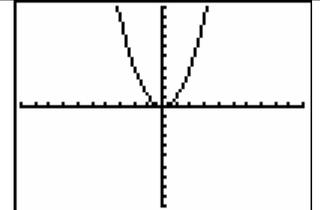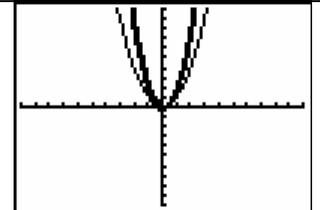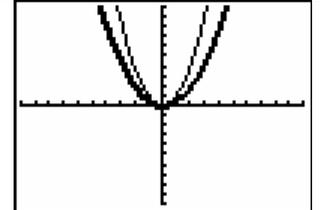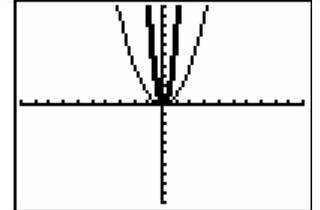
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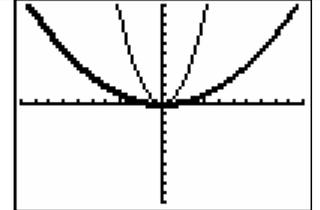
Student Page/Handout
Activity 6: Transformations of Functions (Graphing Calculator Version)

Launch
An understanding of transformations of functions can help you quickly sketch a function. Think about a linear function. Graph the parent function f(x) = x.

    a)   What is the y-intercept for the parent function?

    b)   Write three linear equations that have a different y-intercept? What did you change in your equation?

    c)   What kind of transformation is this?

Throughout this activity you will be looking at the transformations you have already investigated, but now looking at them through function notation.

Explore
1.   Graph the parent function, $f(x) = x^2$ on a zoom standard window.

    a)   Enter $g(x)$ = x^2 + 4 and graph. What happened?

    b)   Enter $h(x)$ = x^2 + 7 and predict what will happen to the parabola. Then graph.

    c)   Enter $j(x)$ = x^2 +2 and predict what will happen to the parabola. Then graph.

    d)   Enter $k(x)$ = x^2 - 2 and graph.

    e)   Enter $p(x)$ = x^2 - 8 and predict what will happen to the parabola. Then graph.

    f)   Enter $r(x)$ = x^2 - 4 and predict what will happen to the parabola. Then graph.

    g)   What can you say about the graphs of the parent function $f(x) = x^2$ and the function $f(x) = x^2 + k$? Name the transformation.

2. Now clear out y_2 to y_7 (g(x) through r(x)).

    a)   Enter $g(x) = (x+4)^2$ and graph. What happened?

    b)   Enter $h(x) = (x+7)^2$ and predict what will happen to the parabola. Then graph.

    c)   Enter $j(x) = (x+2)^2$ and predict what will happen to the parabola. Then graph.

    d)   Enter $k(x) = (x-2)^2$ and predict what will happen to the parabola. Then graph.

    e)   Enter $p(x) = (x-8)^2$ and predict what will happen to the parabola. Then graph.

    f)   Enter $r(x) = (x-4)^2$ and predict what will happen to the parabola. Then graph.

g) What can you say about the graphs of the parent function $f(x) = x^2$ and the function $f(x) = (x - h)^2$? Name the transformation.

3. Now clear out y_2 to y_7 (g(x) through n(x)).

a) Enter $y_2 = g(x) = (x - 4)^2 + 5$ and predict what will happen to the parabola. Then graph. Clear out y_2.

b) Enter $g(x) = (x - 4)^2 - 5$ and predict what will happen to the parabola. Then graph. Clear out y_2.

c) Enter $g(x) = (x + 4)^2 + 5$ and predict what will happen to the parabola. Then graph.
Clear out y_2.

d) Enter $g(x) = (x + 4)^2 - 5$ and predict what will happen to the parabola. Then graph.

e) What can you say about the graphs of the parent function $f(x) = x^2$ and the function $f(x) = (x - h)^2 + k$? Name the transformations.

4. Change the parent function to $f(x) = |x|$ (Absolute value is found under Math, Num, Abs on TI calculators.) Using several different values of h and k describe the transformations between the parent function $f(x) = |x|$ and $g(x) = |x - h| + k$.

5. Let $y_1 = f(x) = x^2$. Think about the graph. Then graph.
   a) Enter $y_2 = g(x) = -x^2$ and predict. Then graph.
   b) Enter $y_3 = h(x) = -x^2 + 7$ and predict. Then graph.
   c) Enter $y_4 = j(x) = -x^2 - 2$ and predict. Then graph.
   Clear out y_2 and y_4.
   d) Enter $y_2 = g(x) = -(x + 7)^2$ and predict. Then graph.
   e) Enter $y_3 = h(x) = -(x - 2)^2$ and predict. Then graph.
   f) What transformation(s) are occurring in (a)- (d)?

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6. Graph $y_1 = f(x) = x^2$ on a zoom standard window.

    a) Enter $g(x) = 2x^2$ and graph.

    b) Enter $h(x) = 4\ x^2$ and predict what will happen to the parabola. Then graph.

    c) Enter $j(x) = 6\ x^2$ and predict what will happen to the parabola. Then graph.

    d) Enter $k(x) = (1/2)\ x^2$ and graph.

    e) Enter $p(x) = (1/3)\ x^2$ and predict what will happen to the parabola. Then graph.

    f) Enter $r(x) = (1/5)\ x^2$ and predict what will happen to the parabola. Then graph.

    g) What can you say about the graphs of the parent function $f(x) = x^2$ and the
        function $f(x) = ax^2$? Name the transformation if a>1, if 0<a<1, and if a = -1.

7) Now clear out y_2 to y_7 Graph $f(x) = |x|$.

    a) Enter $g(x) = 2|x|$ and graph.

    b) Enter $h(x) = 4|x|$ and predict what will happen. Then graph.

    c) Enter $j(x) = 6|x|$ and predict what will happen. Then graph.

    d) Enter $k(x) = \dfrac{|x|}{2}$ and graph.

    e) Enter $p(x) = \dfrac{|x|}{3}$ and predict what will happen.. Then graph.

    f) Enter $r(x) = \dfrac{|x|}{5}$ and predict what will happen.. Then graph.

    g) What can you say about the graphs of the parent function $f(x) = x^2$ and the
        function $f(x) = a|x|$? Name the transformation if a>1, if 0<a<1, and if a = -1.

8) Now clear out y_2 to y_7 Graph $f(x) = x^3$.

    a) Enter $g(x) = (2x)^3$ and graph.

    b) Enter $h(x) = (4x)^3$ and predict what will happen. Then graph.

    c) Enter $j(x) = (6x)^3$ and predict what will happen. Then graph.

    d) Enter $k(x) = \left(\dfrac{x}{2}\right)^3$ and graph.

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e) Enter $p(x) = \left(\dfrac{x}{3}\right)^3$ and predict what will happen. Then graph.

f) Enter $r(x) = \left(\dfrac{x}{5}\right)^3$ and predict what will happen. Then graph.

---

g) What can you say about the graphs of the parent function $f(x) = x^2$ and the function $af(x)$? Name the transformation if a>1, if 0<a<1, and if a = -1.

---

h) What can you say about the graphs of the parent function $f(x) = \sqrt{x}$ and the function $f(bx)$? Name the transformation if b>1, if 0<b<1 and if b=-1.

Summarize

1. Given c>0, describe the transformation of f(x)

a) f(x+c)
b) c*f(x)
c) f(cx)

2. Given c<0, describe the transformation of f(x)
d) f(x+c)
e) c*f(x)
f) f(cx)

3. Given 0<c<1, describe the transformation of f(x)
g) c*f(x)
h) f(cx)

Check for Understandin

Sketch the following graphs without the use of a calculator.

---

a) $f(x) = x^2 + 2$ b) $f(x) = (x+2)^2$ c) $f(x) = -|x-5|$

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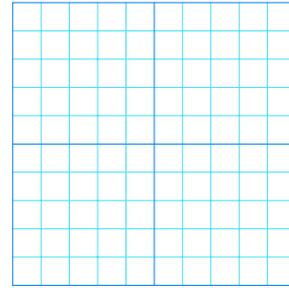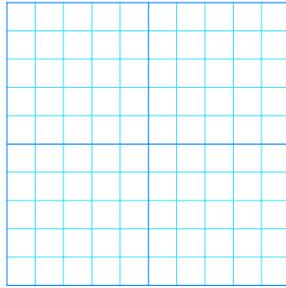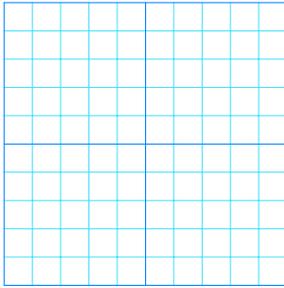
d). $f(x) = |x + 3|$ e). $f(x) = -x^2 + 2$ f). $f(x) = -|x - 3| + 6$

Given the following graphs, write the appropriate y = entries.

A) _____   B) _____
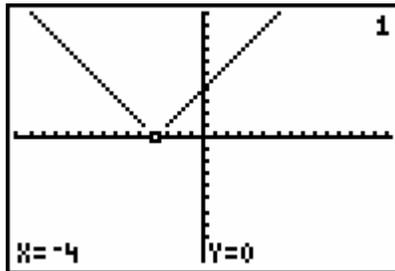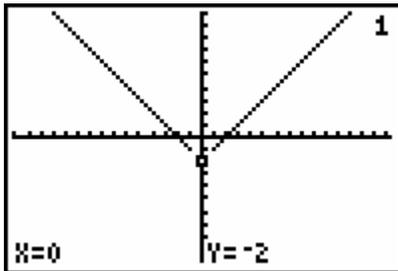
C) _____   D) _____
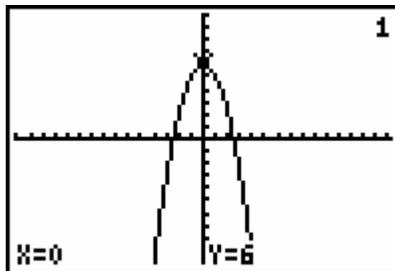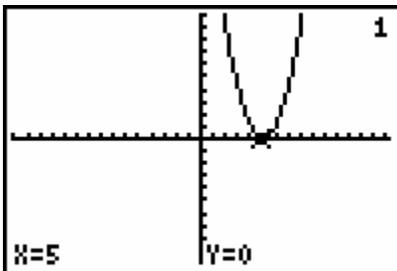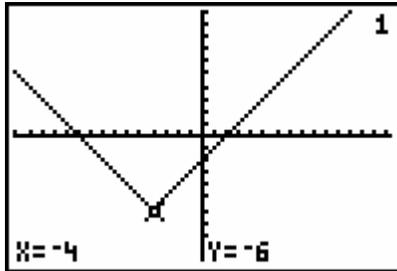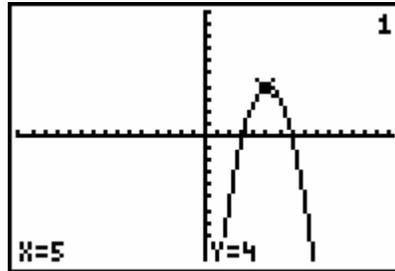
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E)_____   F)_____

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Your instructor will give you a parent function. Complete the following chart for your assigned function.

Function	Describe Translation	***SKETCH IT!***
$y = f(x)$		
$y = f(x+3)$		
$y = f(x-3)$		
$y = f(x)-5$		
$y = f(x)+5$		

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Function	Describe Translation	***SKETCH IT!***
$y = f(x)$		
$y = -f(x)$		
$y = f(-x)$		
$y = 2f(x)$		
$y = \dfrac{f(x)}{2}$		
$y = f(3x)$		
$y = f(\dfrac{x}{3})$		

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Activity 6 Student Page/Handout

Transformations of Functions
Computer Version

Launch
An understanding of transformations of functions can help you quickly sketch a function. Think about a linear function. Graph the parent function f(x)=x.

    a) What is the y-intercept for the parent function?

    b) Write three linear equations that have a different y-intercept? What did you change in your equation?

    c) What kind of transformation is this?

Throughout this activity you will be looking at the transformations you have already investigated, but now looking at them through function notation.

Exploration
2.   Graph the parent function, $f(x) = x^2 + k$.

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

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a) Use the slider to explore the graph for different values of k. How does the value of k effect the graph?

b) What transformation is occurring?

3. Graph the parent function, $f(x) = (x - h)^2$.

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

a) Use the slider to explore the graph for different values of h. How does the value of h effect the graph?

b) What transformation is occurring?

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4. Graph the parent function, $f(x) = (x - h)^2 + k$.

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

a) Use the slider to explore the graph for different values of h and k. How do the values of h and k effect the graph?

b) What transformations are occurring?

5. Graph the parent function $f(x) = |x|$. Now graph $g(x) = |x - h| + k$. Using the sliders for h and k, compare f(x) and g(x).

6. Graph the function $f(x) = ax^2$

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

a) Use the slider to compare the graph for positive values of a ($a>0$) and negative values of a ($a<0$). How does the sign of a affect the graph?

b) Describe the transformation in part (a)?

c) Use the slider to compare the graph for values of a between 0 and 1 ($0< a <1$) and values of a greater than 1 ($a>1$). What happened?

d) Describe the transformation in part (c)?

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7. Graph the function $f(x) = (bx)^3$

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

a) Use the slider to compare the graph for positive values of **b** (**b**>0) and negative values of **b** (**b** <0). How does the sign of **b** affect the graph?

b) Describe the transformation in part (a)?

c) Use the slider to compare the graph for values of **b** between 0 and 1 (0< **b** <1) and values of **b** greater than 1 (**b**>1). What happened?

d) Describe the transformation in part (c)?

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ACTIVITY 7

Activity 7 Overview: Transformations in Statistics

Iowa Core Curriculum Essential Skills/Concepts:	° Understands descriptive statistics ° Understands transformations ° Understands inferential statistics				
Big Ideas	° Analysis of data should include the choice of appropriate representation, the study of measures of center and variability, transformations of univariate data, shape of distributions, outliers, regression, and correlation.				
Characteristics of Instructional Core		Important core content that is evidence-based	x	Deep conceptual and procedural knowledge	x	Reflective teaching in a collaborative setting
	x	Teaching for understanding		Rigorous curriculum		Development of global perspective
	x	Teaching through problem solving and inquiry	x	Relevant curriculum	x	Incorporation of current technology
	x	Student-centered classrooms	x	Assessment for learning		
Cognitive Domain	x	Remembering	x	Applying		Evaluating
	x	Understanding	x	Analyzing		Creating
Connections to Students' Lives		Isolated within discipline		Connected to other disciplines	x	Connected to student lives
		Connected within discipline		Has value beyond school purposes		
Support for Literacy		Using the literacy process for inquiry		Increasing reading volume		
		Increasing access to print		Engaging students with texts		
	x	Involving students in discussion		Reading aloud in content areas		
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension		
		Writing to learn across content areas				
Class Time	1 class period(s) 50 total minutes				

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Materials & Set-up:
Graphing calculators or computer software to complete some statistical calculations
Student handouts

Teaching Tips:
It is probably best to do the launch for this activity the day before the class is to complete it. Then, students can gather data as homework the night before. Also, teachers may want to have data from previous classes or mock data in case someone does not collect their own. The prerequisite knowledge in statistics may vary for different students. Teachers may need to review the statistics information or adapt it for their class of students.

Teacher Tasks	Student Tasks
• Facilitate the launch a day ahead of when the activity will be completed	• Formulate a question with a numerical answer
• Create sample data in case some students do not complete the data collection independently	• Collect data from at least 20 peers.
• Facilitate student's completing the student handout	• Complete the student handout thoughtfully
• Provide a mini summary after each part of the activity to ensure understanding	• Record important notes from any minisummary
• Facilitate the summary	• Record notes and contribute to the summarize.

Differentiation Tasks

Less-Than-Proficient
Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction:
- vocabulary: translate, reflect, measures of center, measures of variability, standard deviation
- skills/concepts: The key is when a translation (adding) is applied to a set of data, the measure of center changes by that amount and the measure of variability does not change. Yet, for a dilation (multiplication), both measure of center and measure of variability changes.

Supplemental instruction suggestions:
- This task can be paced for all students to be successful. Students have choices about their question.
- May want to create a review sheet as there is prerequisite vocabulary and skills necessary in the successful completion of the unit.

Highly Proficient
Supplemental instruction suggestions:
- Giving students freedom to accelerate through the activity will help the highly proficient. Working as a whole class may be frustrating.
- Allowing more freedom about the type of question asked may be enriching for the students.

- Have students investigate on the web further transformations in statistics. There are rich connections beyond the ones indicated in this activity.

Assessment:

Formative Assessments:
- Conducting mini-summaries after each part will be a good indicator of student understanding. It is critical to question without telling to get an accurate idea of student understanding.
- The student responses to the summary indicates the level of student understanding.

Summative Assessments:
- The Check for Understanding should serve as a form of a summative assessment. Activity 8 will not serve well as a summative assessment over these concepts.

Reflect:

The major roadblock is uncovering past student knowledge about statistics. The activity served as a good review of these skills as well as a connection for students. Students were surprised by the way the transformations effected the data at first, but after thinking deeply, they made the connections. They indicated this was one of their favorite activities of the unit. It may be because it is so straightforward.

Activity 7
Student Page/Handout

Transformations in Statistics

Launch

With your class, brainstorm some questions. You could ask about the time spent on different activities or events during their day. These questions must be able to be answered numerically. For example, "How many minutes a day on average do you spend doing homework?" Decide on an activity or event you would like to explore more in depth. Collect data from at least 20 peers.

Exploration

Part A
You will need to use your collected data to complete the following questions.

1. Make a quick dot plot of your data. Describe the shape of your plot.

2. Find the five number summary for your date. Recall this includes minimum, first quartile, median, third quartile, and maximum.

3. Quickly make a box plot with the information from #2.

4. Find the two additional measures of central tendency: the mean and mode.

5. Find the standard deviation for your data.

6. Write a brief description of your collected data.

Part B

Now, let's take a look at how the data and statistics would change if your classmates responded a little differently. Let's say that when you collected the data you asked about time spent on homework. Then, your classmates all pledge to spend just 5 more minutes on homework each day.

1. How would your dot plot change? What would stay the same?

2. How would your five number summary change?

3. How would your box plot change? What would stay the same?

4. Would the mean and/or the mode change? Why or why not?

5. Would the standard deviation change? Why or why not?

6. How would you need to revise your description of your data?

7. What type of transformation is this? Why?

8. Briefly describe what would be the same and different about your data if all your classmates spent five fewer minutes on the activity?

Part C

Again, let's take a look at how the data and statistics would change if your classmates responded by saying they are going to double the amount of time they spend on the activity. If they have been spending 30 minutes a day on homework they would now be spending 60 minutes a day. Consider the same questions above.

1. How would your dot plot change?

2. How would your five number summary change?

3. How would your box plot change? What would stay the same?

4. Would the mean and/or the mode change? Why or why not?

5. Would the standard deviation change? Why or why not?

6. How would you need to revise your description of your data?

7. What type of transformation is this? Why?

8. Briefly describe what would be the same and different about your data if all your classmates spent half as much time on the activity?

Summarize

1. If a value c is added or subtracted to elements of the data set, what happens to the measures of central tendency? What happens to the measures of variability? Why?

2. If a value c is multiplied by each element of the data set, what happens to the measures of central tendency? What happens to the measures of variability? Why?

3. This activity discussed translations and dilations for a data set. Do you think there is a rotation of a set of data or a reflection of a set of data? Why or why not?

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ACTIVITY 8

Activity 8 Overview: *Mini-animation*

Iowa Core Curriculum Essential Skills/Concepts:	° Understands and applies functions. ° Understands and applies coordinates ° Understands and applies transformations ° Communicates his/her mathematical thinking coherently and clearly to peers, teachers, and others ° Uses the language of mathematics to express mathematical ideas precisely ° Uses representations to model and interpret physical, social, and mathematical phenomena ° Selects, applies, and translates among mathematical representations to solve problems ° Creates and uses representations to organize, record, and communicate mathematical ideas
Big Ideas	° Coordinates model points, lines, and geometric shapes. These help one analyze and reason about properties including slope, distance, midpoints, and perpendicularity. ° Coordinate methods, matrices, and technology model composition of transformations. ° Transformations of objects can be visualized and described. ° Algorithmic thinking (input, process, output) is used to develop calculator programs for slope, distance and midpoint and animations illustrating. ° Communicating understanding of coordinates and transformations involves specific (new to some) vocabulary.

Characteristics of Instructional Core		Important core content that is evidence-based	x	Deep conceptual and procedural knowledge	x	Reflective teaching in a collaborative setting
	x	Teaching for understanding	x	Rigorous curriculum		Development of global perspective
	x	Teaching through problem solving and inquiry	x	Relevant curriculum	x	Incorporation of current technology
	x	Student-centered classrooms	x	Assessment for learning		

Cognitive Domain	x	Remembering	x	Applying		Evaluating
	x	Understanding	x	Analyzing	x	Creating

Connections to Students' Lives		Isolated within discipline	x	Connected to other disciplines	x	Connected to student lives
	x	Connected within discipline		Has value beyond school purposes		

Support for Literacy		Using the literacy process for inquiry		Increasing reading volume
		Increasing access to print		Engaging students with texts
		Involving students in discussion		Reading aloud in content areas
		Increasing reading fluency		Explicitly instructing in vocabulary and comprehension
		Writing to learn across content areas		

Class Time	2 class period(s) approximately 100 total minutes

Materials & Set-up:
Computers for each student with CPMPTools http://www.wmich.edu/cpmp/CPMP-Tools/ or graphing calculators for all students
Student handout
Any of the additional aids supplied at the end of the activity that would be helpful for your students.

Teaching Tips:
There have been many lessons which use a version of this idea including hand flip books, calculator program animations, calculator movements (non-programming), Geometer's Sketchpad, LOGO, etc. The majority of the materials listed here use CPMP-Tools software. There are many ways to make it successful. Attached to the end of this document is a rubric, a scaffolded programming activity, and a sharing day handout. This activity can be worked on after Activity 5. It should be introduced early so students have time to work on it outside of class. Minimal class time should be needed, but it is necessary to allow time for questions and getting comfortable with the technology. Also included in the additional materials are mini-activities labeled translation station, reflection station, and rotation station. These mini-activities may be helpful if your students will be using the computer software program. The level of rigor is not exceedingly high for mini-activities, but can serve as building blocks to a successful animation.

Teacher Tasks	Student Tasks
- Introduce the project after Activity 5. Begin familiarizing students with the technology. - Give feedback on initial objects to be animated. - Provide time inside and outside of class for questions about technology and visualizing the transformations - Set aside a portion of one class to have students collaborate. - Provide a safe structure for students to share work. Question students about their understanding	- Create an initial object to be animated. This can be done on graph paper first and then inputted or done on a calculator or computer. - Investigate ways to make your object move using transformations and matrices. Students will need to review notes. - Begin animating object and creating the code or flip book. - Highlight transformations and coordinate representations. - Share animation with other students for feedback and improvement. - Share animation with class using formal mathematical language.
Differentiation Tasks	

Less-Than-Proficient
Prioritized vocabulary, skills, or concepts for supplemental or intensive instruction:
Skills/concepts: Students need to build a visual understanding of transformations as well as being able to represent them in multiple ways.

Supplemental instruction suggestions:

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- This task can be paced for all students to be successful. Students have choice in the complexity of the animations.
- May want to create a "cheat sheet" for desired codes to complete particular transformations.
- Set up a discussion board online for student's to pose questions.

Highly Proficient
Supplemental instruction suggestions:
- This task can be self-differentiating for students as many highly proficient students can challenge themselves to complete more complex animations.
- Question students about how to complete more complex animations and challenge them to include them in their animations projects.

Assessment:

Formative Assessments:
- Students often try particular transformations unsuccessfully. Providing scaffolded questions will help them build understanding of how to get their objects to move accurately.
- The collaboration day is a quick way of getting feedback on students' progress.

Summative Assessments:
- This project can be used as a summative assessment for many of the concepts from Activities 1-5.

Reflect:

This project has had a wide degree of success depending upon a student's tolerance for detail work and visualization. It has served to connect some representations of transformations and engage students meaningfully. As a result of the activity, one of the most frequent learning outcomes has been increased visualization reasoning. Students typically think they know what will happen when using a particular transformation, but often are surprised at the true path. This makes students predict and revise frequently. Students typically remember this project several years after it has been completed.

Activity 8
Student Page/Handout

Animate This!

Assignment: Create a short animation. Create a picture that interests you. This picture can have multiple objects. Perform at least five different transformations on that shape to create a short animation. Repeated transformations are great for animation effect, but will only qualify for one of your five transformations.

For this project you will share your understandings and products in three ways:
- Hand in a copy of the programming code or sketchpad script or calculator program for your animation program. Highlight all of the transformations included on the code.
- For a single point on your shape, create a coordinate mapping for each transformation that occurs in your animation. For the point follow it throughout the entire animation.
  ex: $(x,y) \longrightarrow (x+3, y+4)$.
- Share the animation with the class. During this time, please identify the transformations on the screen.

SUPPLEMENT TO ACTIVITY 8

CONSULTANT DAY FOR ANIMATE THIS! PROJECT

Goals: Share your draft of your animation with your consultant group. Listen to each other carefully as well as watching the animation (if it is ready). Then, give each other feedback to make your projects better.

In the space below record some examples of feedback you received on your project, both positive and constructive.

In the space below record some examples of feedback you gave to others in your group, both positive and constructive.

Rate your effort and effectiveness today in being a consultant to others in your group out of 10.   _____/10
Explanation:

Instructor rating for your effort and effectiveness today in being a consultant to others in your group.   _____/10
Comments:

Coordinate Mapping Animation Project Evaluation Rubric

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	Beginning Understanding	Developing Understanding	Strong Understanding	Exceptional Understanding
Original Shape	Single shape of a simple object ☐	Simple objects, but more than one created ☐	Object is somewhat complex and/or more than one object created for the animation ☐	Several complex objects created as part of the animation ☐
Animation Program	Only one or two types of transformations are used repeatedly to create a very basic animation ☐	Several transformations are used, but are less than five total or create an animation that lacks purpose. ☐	Five transformations create a creative, purposeful animation ☐	More than five transformations create an especially interesting, creative, and complex animation. ☐
Coordinate Mapping	No given coordinate rules for transformations in animation, or all rules are incorrect. ☐	Some coordinate rules are given for transformations, many given are correct. ☐	Correct transformation rules are given for each transformation. ☐	Correct transformation rules are given for every transformation, including complex compound rules. ☐
Overall Mathematical Accuracy	Many errors in programming and coordinate rules. ☐	Some of the programming and coordinate rules are correct. ☐	Nearly all programming and coordinate rules are correct ☐	All programming and coordinate rules are correct ☐

COMMENTS:

Supplement to Activity 8

Sample Mini-Program Activity using CPMP-Tools http://www.wmich.edu/cpmp/CPMP-Tools/

Open CPMP-Tools to Geometry: Coordinate Geometry. Be sure Course 2 is selected from the main screen. Find and open command window at the bottom of the screen. Ask if you cannot find it. Try the following short programs. Be careful to include each space and each bracket. Analyze what the programming code says compared to what happens on the computer. You can copy and paste into command window. The HELP menu can be used as well.

ROTATE SHUTTLE

let shuttle = [[8,0][2,0][2,4][0,4][0,8][0,4][-2,4][-2,0][-8,0][8,0]]
draw shuttle
repeat 8 [draw [let shuttle = [rotate shuttle 45]] pause 500]

Describe what happens:

SCALE SHUTTLE

let shuttle = [[8,0][2,0][2,4][0,4][0,8][0,4][-2,4][-2,0][-8,0][8,0]]
draw shuttle
repeat 4 [draw [let shuttle = [scale shuttle 1.5]] pause 500]

Describe what happens:

Translate Shuttle
let shuttle = [[8,0][2,0][2,4][0,4][0,8][0,4][-2,4][-2,0][-8,0][8,0]]
draw shuttle
repeat 4 [draw [let shuttle = translate shuttle [2,3]] pause 500]
fill shuttle 200 0 200

Describe what happens after the first three lines. What does the fourth line do?

Flag Animation
let flag = [[4,0][4,8][6,6][4,4]]
draw flag

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```
draw [let image = [rotate flag 45]]
pause 500
hide flag
draw image
pause 500
repeat 16 [draw [let image = [rotate image 45]] pause 100]
pause 500
let reflect = [[-1,0][0,1]]
let image = [reflect*image]
hide image
pause 500
repeat 16 [draw [let image = [rotate image 315]] pause 100]
```

SUPPLEMENT TO ACTIVITY 8

SHARING MINI-PROGRAMS   PREDICT-ANALYZE-EXTEND

Below is the programming code for your classmate's animation project. As a group, predict what you think will happen when your classmate shares his/her project. Your prediction should be specific for example instead of "an object will be translated", say, "an object will be translated 5 units to the right and 3 units down."

Code:
```
let flower=
[[5,0][5,3][6,2][7,3][6,4][7,5][6,6][5,5][4,6][3,5][4,4][3,3][4,2][5,3]]
draw flower
let flowerB = [scale flower 2]
let flowerB = [translate flowerB [-5,0]]
pause 200
hide flower
draw flowerB
pause 200
let flowerC = [scale flower 3]
let flowerC = [translate flowerC [-10,0]]
[pause 200]
hide flowerB
draw flowerC
let petal = [[2,18][-1,15][2,12][5,15][2,18]]
hide flowerC
let flower-p = [[5,0][5,3][6,2][7,3][6,4][7,5][6,6][3,3][4,2][5,3]]
let flower-p =  [scale flower-p 3]
let flower-p = [translate flower-p [-10,0]]
draw flower-p
repeat 11 [draw [let petal = [rotate petal 10]] [pause 100]]
hide flower-p
hide petal
let petal2 = [[8,12][11,15][8,18][5,15][8,12]]
let flower-2 = [[5,0][5,9][8,6][11,9][8,12][5,15][-1,9][2,6][5,9]]
draw flower-2
repeat 11 [draw [let petal2 =  [rotate petal2 10]]pause 100]
hide flower-2
hide petal2
let petal3 = [[-1,9][2,6][5,9][2,12][-1,9]]
let flower-3 = [[5,0][5,9][8,6][11,9][5,15][2,12][5,9][5,0]]
draw flower-3
repeat 11 [draw [let petal3 = [rotate petal3 10]]pause 100]
hide flower-2
hide petal3
let petal4 = [[5,9][8,6][11,9][8,12][5,9]]
let flower-4 = [[5,0][5,9][8,12][5,15][2,12][5,9][5,0]]
draw flower-4
```

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```
hide flower-3
draw petal4
repeat 13 [draw [let petal4 = [rotate petal4 10]]pause 100]
hide petal4
```

Prediction by group (including all mathematical transformations):

After the transformation is shared:

1) Was your group correct? If not, what should have been changed about your prediction?

2) How could you extend the animation of your classmate?

3) What is the series of coordinate mappings that would give the composite transformation for the first three transformations in the animation?

(x,y) $\longrightarrow$ (____, _____) $\longrightarrow$ (____, _____) $\longrightarrow$ (____, _____)

Supplement to Activity 8

Translation Station

First Experience: Graph Paper (Coordinate representation)
1) On graph paper below **translate** the triangle 3 units to the right and 2 units down. Label the new triangle A', B', and C' respectively.
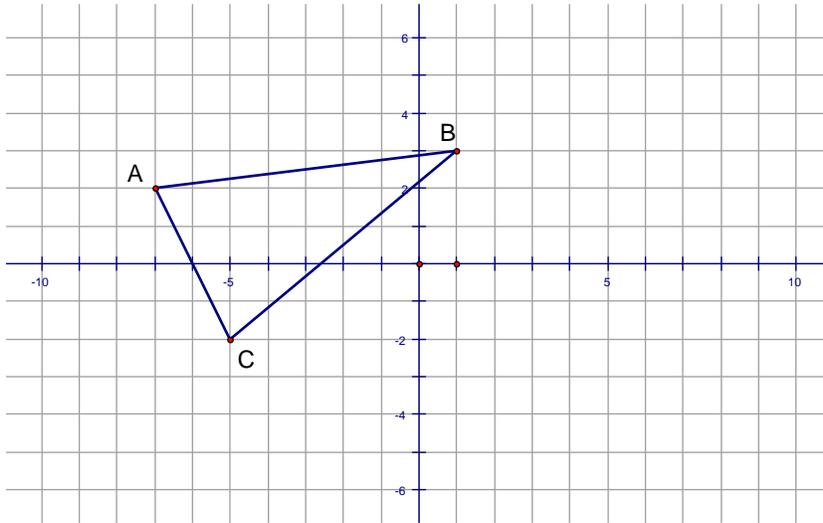
2) Below is a **translation** of triangle ABC. How would you specifically describe it?
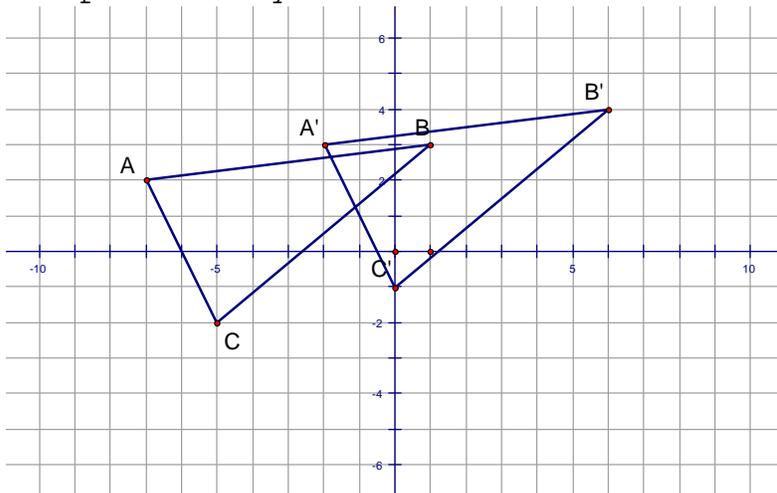
3) One way people represent a translation is using a coordinate rule. The translation in #1 can be described using a coordinate rule: $(x,y)\longrightarrow(x+3,y-2)$. Use a coordinate rule to represent the translation in #2:
$(x,y)\longrightarrow(\underline{\qquad},\underline{\qquad})$.

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Second Experience: Computer (programming language)
Open GeoTools (be sure under the Course menu you are in Course Two) and open the command window at the bottom of the screen.

1) Go to Custom Tools and choose human.
2) Translate the human. To do this click the 2nd icon from the right, then click and drag the human until you have found any new position for it.
3) How many units did it move in the horizontal direction?

4) How many units did it move in the vertical direction?

5) Now it is your turn to try. There is a quick way to make your human move vertically and horizontally using programming language as well.
    • Clear the screen by typing cs (return)
    • Go to sample animations and choose humanoid.
    • Type: draw [let humanoid = [translate humanoid [-2, -3]]]
    What happened?

6) Move the human using your own translation. Record your line of code
    below.

Third Experience: Matrices (matrix representation)
You can also represent a translation using matrices. Let the first matrix represent triangle ABC from the beginning of this activity. What matrix would represent a translation of 3 units to the right and 2 units down?   (The last matrix represents the image matrix.)

$$\begin{bmatrix} -7 & -5 & 1 \\ 2 & -2 & 3 \end{bmatrix} + \begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix} = \begin{bmatrix} -4 & -2 & 4 \\ 0 & -4 & 1 \end{bmatrix}$$

	Definition	Coordinate representation of a translation (h,k)	Matrix	Programming Language

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Translation				

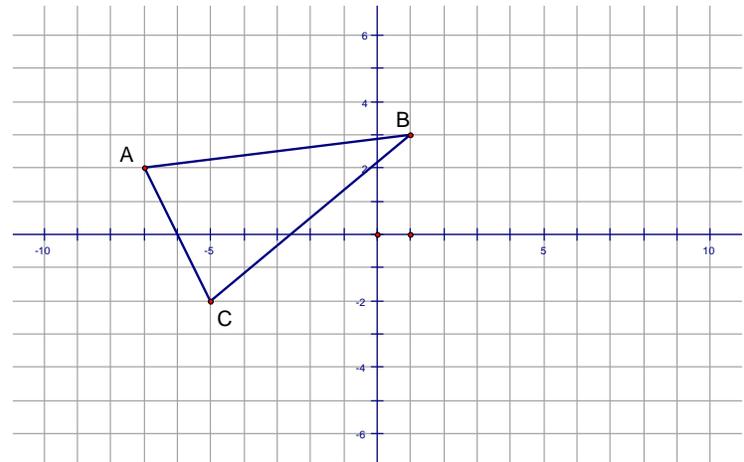## Reflection Station

First Experience: Graph Paper (Coordinate representation)

1) On graph paper below **reflect** the triangle ABC in the **y-axis**. Fill in the blanks below with the new coordinates of the image triangle.

```
PREIMAGE        IMAGE

  A(-7,2) ⟶ A' (_____,
  _____)
  B(1, 3) ⟶ B' (_____,
  _____)
  C(-5, -2) ⟶ C' (____,
  ___)
```

For any point reflected about the y-axis, the coordinate representation   is:
      Preimage   Image
      (x, y) ⟶ (_____, _____)

2) On graph paper below **reflect** the triangle ABC about the **x-axis**. Fill in the blanks below with the new coordinates of the image triangle.

```
PREIMAGE         IMAGE

  A(-7,2) ⟶ A' (_____,
  _____)
  B(1, 3) ⟶ B' (_____,
  _____)
  C(-5, -2) ⟶ C' (____,
  ___)
```

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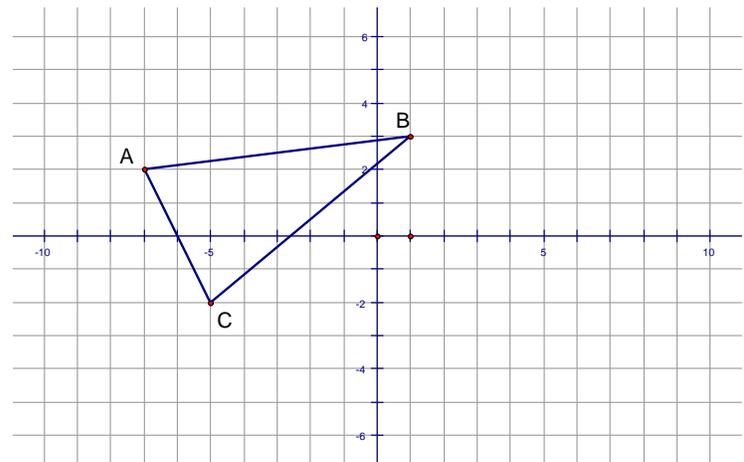
For any point reflected about the x-axis, the coordinate representation is:
    Preimage   Image
    (x, y) ⟶ (_____, _____)

3) Below is a **reflection** of triangle ABC about the line y=x. How would you specifically describe it using coordinates?

PREIMAGE	IMAGE

A(-7,2) ⟶ A' (_____, _____)

B(1, 3) ⟶ B' (_____, _____)

C(-5, -2) ⟶ C' (____, ____)

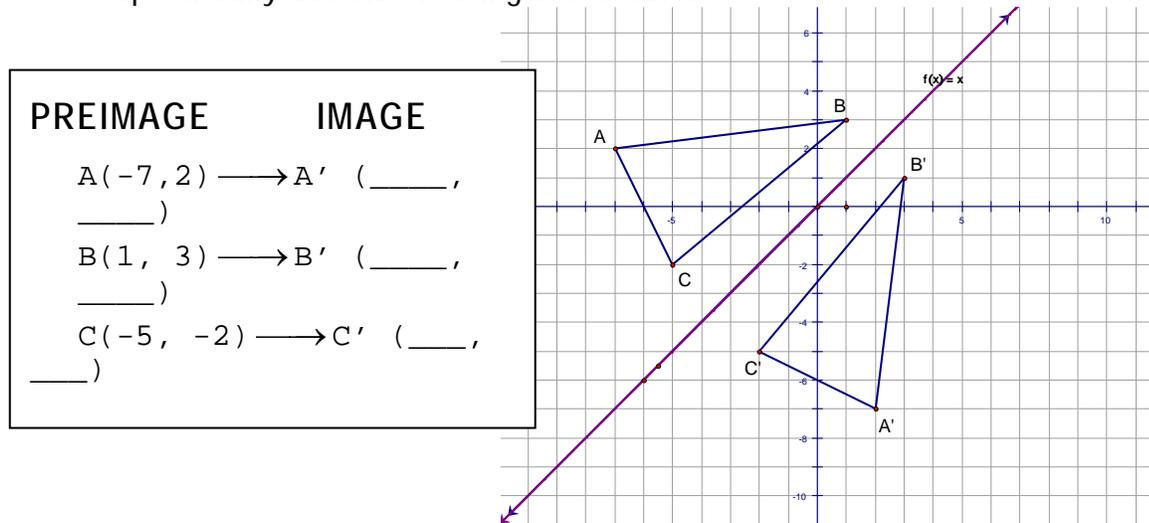**For any point reflected in the y=x, the coordinate representation is:**
    Preimage   Image
    (x, y) ⟶ (_____, _____)

Second Experience: Matrix representation
    1) Matrices can be used to represent reflections as well. To simplify this let's reflect just point A. We know that A (-7,2) is reflected about the y-axis to A' (7, 2). We need to find the 2x2 matrix that when we multiply it by the preimage matrix we get the image matrix.

We know: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -7 \\ 2 \end{bmatrix} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$

But for a reflection about the y-axis we want

$\begin{bmatrix} \underline{} & \underline{} \\ \underline{} & \underline{} \end{bmatrix}\begin{bmatrix} -7 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$ **What values go in each blank?**

The matrix above should represent a reflection about the y-axis for any point (x, y).

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2) Find the matrix representation for a reflection about the x-axis for any point (x,y) by filling in the missing blanks.

$$\begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$$ **What values go in each blank?**

3) Find the matrix representation for a reflection about the line y=x for any point (x,y) by filling in the missing blanks. (You may want to refer to the reflection work done earlier.

$$\begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \end{bmatrix}$$ **What values go in each blank?**

Third Experience: Computer (programming language)
Open GeoTools (be sure under the Course menu you are in Course Two) and open the command window at the bottom of the screen.

7) Go to sample animation and choose humanoid.
8) Type:
   let reflectmatrix = [[1,0][0,-1]]
   draw [let humanoid = [reflectmatrix*humanoid]]

9) What happened? How does the code match what happened on the computer screen?

10) How can you change the code in #2 to do a reflection about the y-axis? Select a new screen (next to the arrow) and go to custom tools to choose human, again. Try out your new code. Record a working code below.

11) How can you change the code in #2 to do a reflection about the line y=x? Select a new screen (next to the arrow) and go to custom tools to choose human, again. Try out your new code. Record a working code below.

Summarize

	Definition	Coordinate Representation	Matrix Representation	Programming Language
Reflection in y-axis				
Reflection in x-axis				
Reflection in y=x				

Rotation Station

First Experience: Graph Paper (Coordinate representation)

   3) On graph paper below is the Flag ABCDE rotated by 90 degrees
      counterclockwise about the origin several times. Fill in the blanks below with
      the new coordinates of the image triangle for the first 90 degree rotation.

90 DEGREE COUNTERCLOCKWISE
ROTATION ABOUT THE ORIGIN

PREIMAGE   IMAGE

A(0,0) ⟶ A' (_____,
_____)

B(3,1) ⟶ B' (_____,
_____)

C(5,2) ⟶ C' (____,
____)
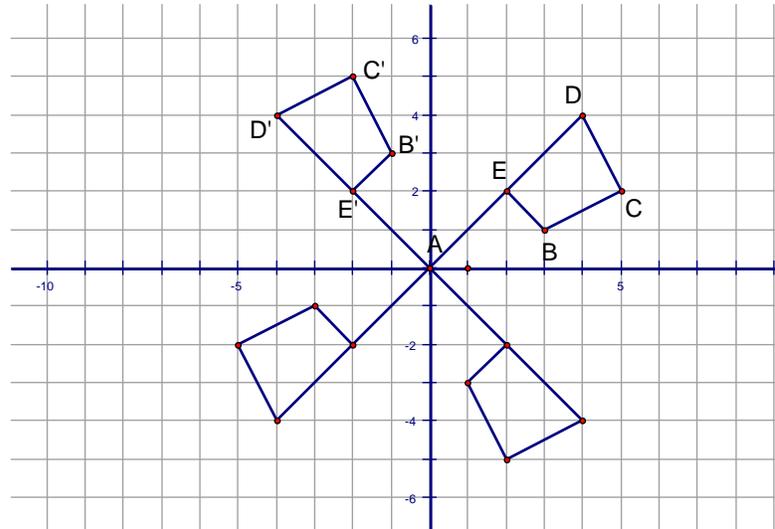
**For any point rotated by 90 degrees (counterclockwise), the coordinate
representation is:**

      Preimage   Image
      (x, y) ⟶ (_____, _____)

2) Using the rotations above find out what the coordinate representation is for a
counterclockwise rotation of 180 degrees about the origin.

180 DEGREE COUNTERCLOCKWISE
ROTATION ABOUT THE ORIGIN

PREIMAGE   IMAGE

A(0,0) ⟶ A' (_____,
_____)

B(3,1) ⟶ B' (_____,
_____)

C(5,2) ⟶ C' (____,
____)

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For any point rotated by 180 degrees (counterclockwise), the coordinate representation is:

　　　Preimage　　　Image

　　　(x, y) $\longrightarrow$ (_____, _____)

3) Using the rotations above find out what the coordinate representation is for a counterclockwise rotation of 270 degrees about the origin.

270 DEGREE COUNTERCLOCKWISE ROTATION ABOUT THE ORIGIN

PREIMAGE　　　　　IMAGE

　A(0,0) $\longrightarrow$ A' (_____,
　_____)
　B(3,1) $\longrightarrow$ B' (_____,
　_____)
　C(5,2) $\longrightarrow$ C' (___, ___)
　D(4,4) $\longrightarrow$ D' (___,
　　　　　　　　）

For any point rotated by 270 degrees (counterclockwise), the coordinate representation is:

　　　Preimage　　　Image

　　　(x, y) $\longrightarrow$ (_____, _____)

Second Experience: Matrix representation

4)   Matrices can be used to represent rotations as well. To simplify this let's rotate just point B. We know that B (3,1) is rotated counterclockwise 90 degrees about the origin to B' (-1, 3). We need to find the 2x2 matrix that when we multiply it by the preimage matrix we get the image matrix.

We know: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

But for a counterclockwise rotation of 90 degrees about the origin, we want

$\begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ **What values go in each blank?**

The matrix above should represent a counterclockwise rotation of 90 degrees about the origin for any point (x, y).

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5) Find the matrix representation for a counterclockwise rotation of 180 degrees about the origin for any point (x,y) by filling in the missing blanks.

$$\begin{bmatrix} \underline{} & \underline{} \\ \underline{} & \underline{} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$ **What values go in each blank?**

6) Find the matrix representation for a counterclockwise rotation of 180 degrees about the origin for any point (x,y) by filling in the missing blanks.

$$\begin{bmatrix} \underline{} & \underline{} \\ \underline{} & \underline{} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \underline{} \\ \underline{} \end{bmatrix}$$ **What values go in each blank?**

Third Experience: Computer (programming language)
7) Go to sample animation and choose humanoid.
8) Type:
   let rotatematrix = [[-1,0][0,-1]]
   draw [let humanoid = [rotatematrix*humanoid]]

9) What happened? How does the code match what happened on the computer screen?

10) How can you change the code in #2 to do a counterclockwise rotation of 90 degrees about the origin? Select a new screen (next to the arrow) and go to custom tools to choose human, again. Try out your new code. Record a working code below.

11) How can you change the code in #2 to do a counterclockwise rotation of 270 degrees about the origin? Select a new screen (next to the arrow) and go to custom tools to choose human, again. Try out your new code. Record a working code below.

Summarize

Counterclockwise rotation of	Definition	Coordinate Representation	Matrix Representation	Programming Language
90 degress				
180 degrees				
270 degrees				

If you finish before other groups, try the following program.

First select a new screen and choose humanoid. Type in the command window:

let rotatematrix = [[-1,0][0,-1]]
draw [let humanoid = [rotatematrix*humanoid]]
repeat 8 [draw [let humanoid = [rotate humanoid 45]] pause 50]

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