Interpreting Functions F-IF

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). (F-IF.1.)

x	y
-2	3
-1	0
0	-1
1	0
2	3
3	8

The table above shows all the ordered pairs (x, y) that define a relation between the variables x and . Is y a function of x?

Yes No

Give a reason for your answer.

2009-12-2-7

Source: National Assessment of Educational Progress, 2009, Grade 12 Mathematics Assessment.

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2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F-IF.2.)

If $f(x) = \frac{2x+1}{3}$ and $g(x) = 2x^2 + 2$, then $f(g(2)) =$

A. 3
B. 5
C. 7
D. $7\frac{5}{9}$
E. $16\frac{2}{3}$

1992-12-5-20

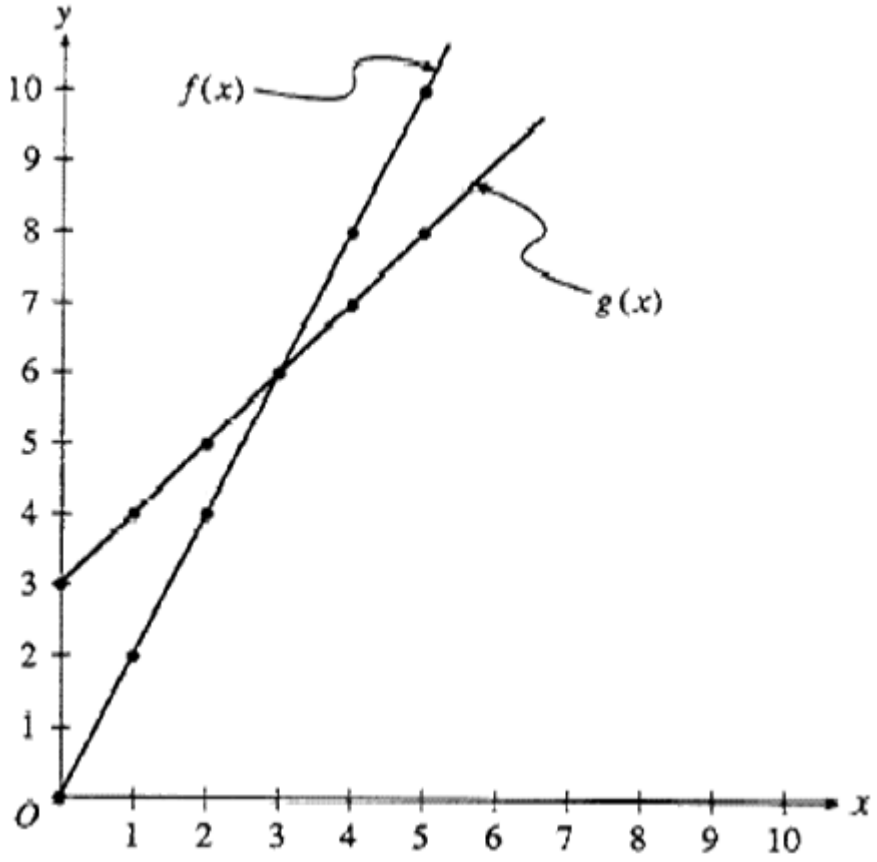
Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

If $f(x) = 4x^2 - 7x + 5$, what is the value of $f(3.5)$?

1992-12-7-9

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

The following question refers to the graph shown below.

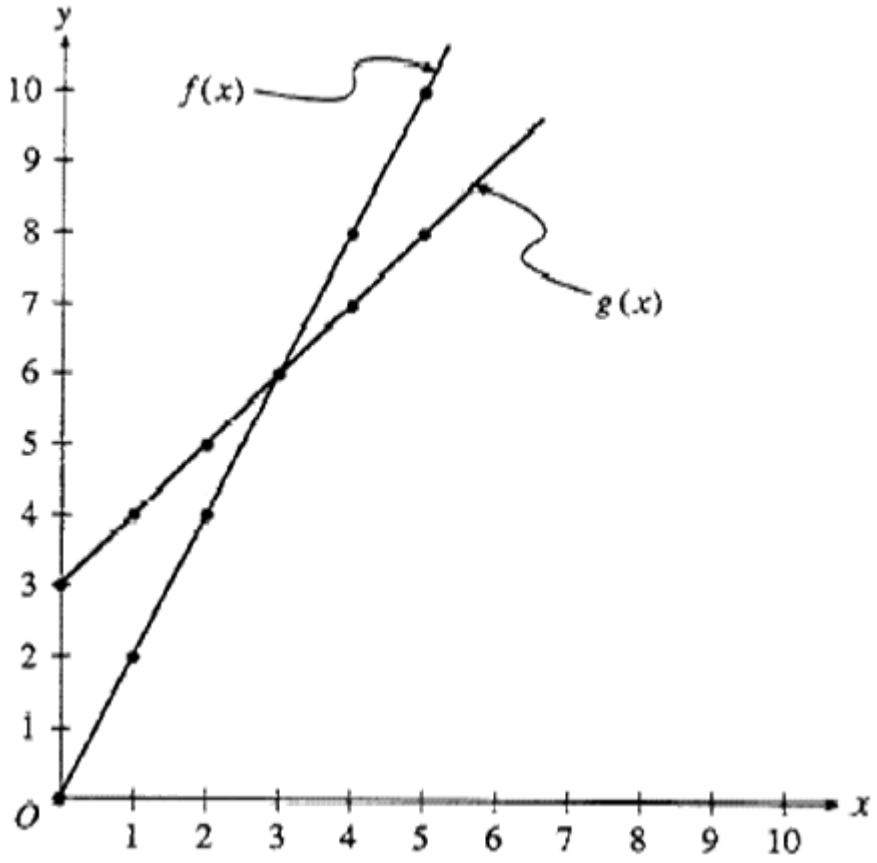


What is the value of $g(1)$?

A. 2
B. 4
C. 5
D. 6
E. 8

1990-12-7-8

Source: National Assessment of Educational Progress, 1990, Grade 12 Mathematics Assessment.



What is the value of $f(g(1))$?

A. 2
B. 4
C. 5
D. 6
E. 8

1990-12-7-19

Source: National Assessment of Educational Progress, 1990, Grade 12 Mathematics Assessment.

If $f(z) = z + 8$, what is the value of $f(6)$?

2008-17-21-23

Source: National Assessment of Educational Progress, 2008, Age 17 Mathematics Assessment.

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3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. (F-IF.3.)

Suppose that $a_1, a_2, a_3, \ldots$ is the sequence of numbers such that $a_1 = 3$, $a_2 = \sqrt{a_1} + 1$, $a_3 = \sqrt{a_2} + 1$, and, in general, $a_{n+1} = \sqrt{a_n} + 1$ for all $n \geqq 1$. To the nearest hundredth, the value of a_5 is

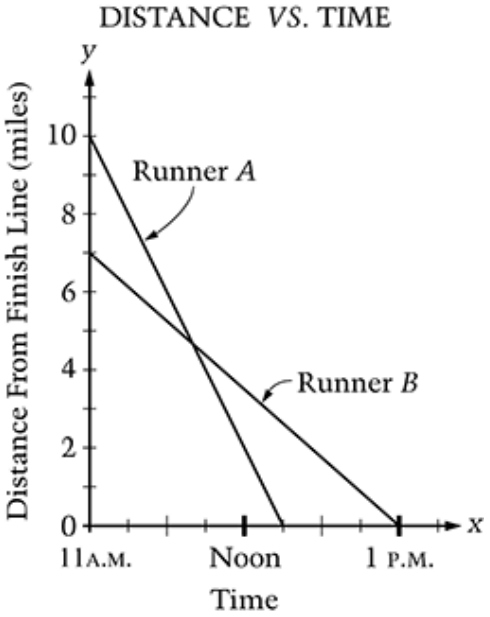
A. 1.63
B. 2.62
C. 2.73
D. 3.24
E. 5.73

1990-12-7-7

Source: National Assessment of Educational Progress, 1990, Grade 12 Mathematics Assessment.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F-IF. 4.)



The graph above shows distance versus time for a race between runners A and B. The race is already in progress, and the graph shows only the portion of the race that occurred after 11 A.M.

The following table lists several characteristics of the graph. Interpret these characteristics in terms of what happened during this portion of the race. Include times and distances to support your interpretation. (A sample interpretation of the y-intercepts is given in the table.)

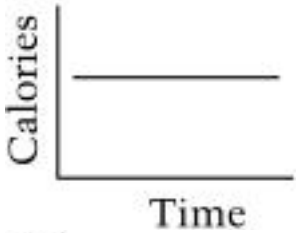
Characteristic of Graph	Interpretation in Terms of the Race
y-intercepts	At 11 A.M. Runner A is 10 miles from the finish line and Runner B is 7 miles from the finish line.
Slopes	
Point of intersection	
x-intercepts	

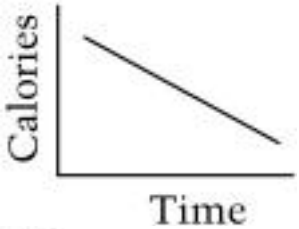
2009-12-2-9

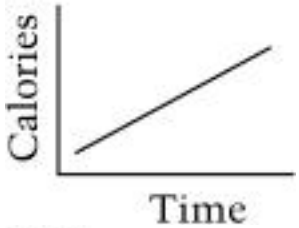
Source: National Assessment of Educational Progress, 2009, Grade 12 Mathematics Assessment.

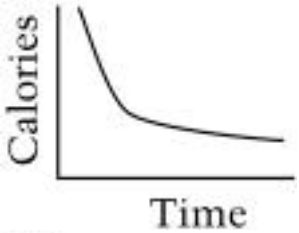
Exercise Time (minutes)	Total Calories Burned
10	37
12	42
14	49
16	58
18	64
20	70

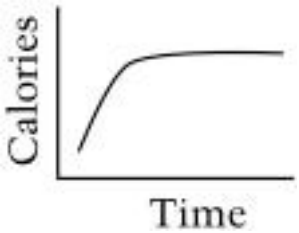
Which of the following graphs best illustrates the relationship between exercise time and total calories burned, as shown in the table above?

A. 

B. 

C. 

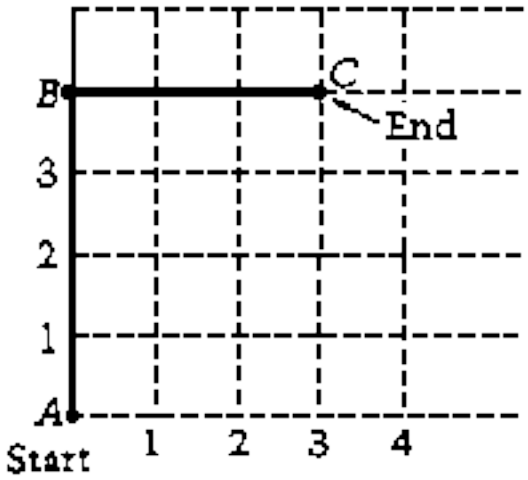
D. 

E. 

2005-12-3-5

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

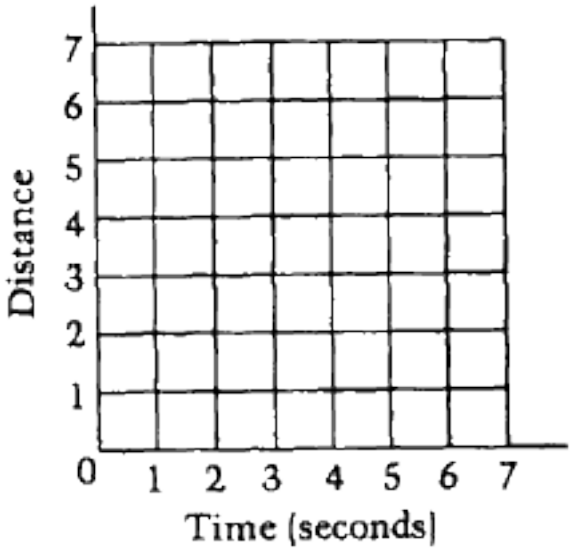


The darkened segments in the figure above show the path of an object that starts at point *A* and moves to point *C* at a constant rate of 1 unit per second. The object's distance from point *A* (or from point *C*) is the shortest distance between the object and the point.

In the space below, complete the following steps.

a) Sketch the graph of the distance of the object from point *A* over the 7-second period.

b) Then sketch the graph of the distance of the object from point *C* over the same period.

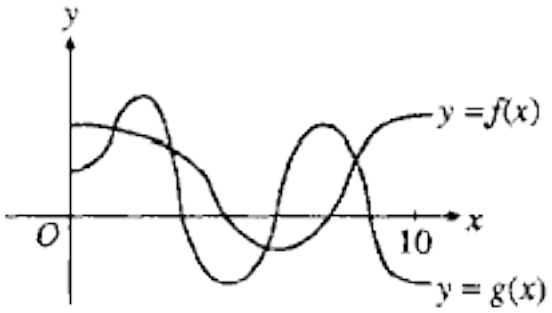


c) On your graph, label point *P* at the point where the distance of the object from point *A* is equal to the distance of the object from point *C*.

d) Between which two consecutive seconds is the object equidistant from points *A* and *C*?

1992-12-15-11

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.



The graphs of y = f(x) and y = g(x) for $0 \le x \le 10$ are shown in the figure above. For how many values of x is the product f (x) g(x) = 0 for $0 \le x \le 10$?

A. Two
B. Four
C. Five
D. Six
E. Seven

1990-12-9-19

Source: National Assessment of Educational Progress, 1990, Grade 12 Mathematics Assessment.

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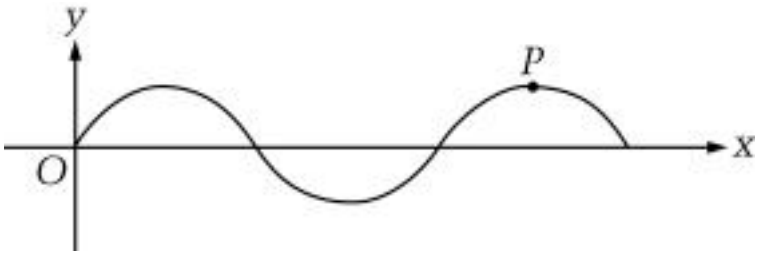
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F-IF.5.)

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
(F-IF.6.)

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (F-IF.7.)



The graph of $f(x) = \sin x$ is shown above. Which of the following is the *x* coordinate of point *P*?

A. $\frac{\pi}{2}$
B. π
C. $\frac{3\pi}{2}$
D. 2π
E. $\frac{5\pi}{2}$

2005-12-4-17

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay. (F-IF.8.)

The population P of a certain town is given by the equation $P = 50{,}000(1+r)^t$, where r is the annual rate of population increase and t is the number of years since 1990.

(a) What was the population in 1990?

(b) In 2001 the population was 100,000. What was the annual rate of population increase?

2009-12-7-3

Source: National Assessment of Educational Progress, 2009, Grade 12 Mathematics Assessment.

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9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F-IF.9.)

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
   c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. (F-BF.1.)

The following chart indicates the maximum number of connecting line segments y that can be drawn connecting x points, where no three points lie on the same line.

Number of points (x)	1	2	3	4	5
Maximum number of connecting line segments (y)	0	1	3	6	10
Example					

(a) The relationship between x and y is represented by the equation $y = kx(x - 1)$ for any positive number of points x . Use the information in the table to determine the value of the real number k .

Answer: k = _____

(b) Use the equation from part (a) to determine the maximum number of line segments that can be drawn connecting 100 points, no three of which lie on the same line.

Answer: _____

2009-12-7-10

Source: National Assessment of Educational Progress, 2009, Grade 12 Mathematics Assessment.

If $f(x) = x^2 + x$ and $g(x) = 2x + 7$, what is an expression for $f(g(x))$?

2005-12-3-16

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

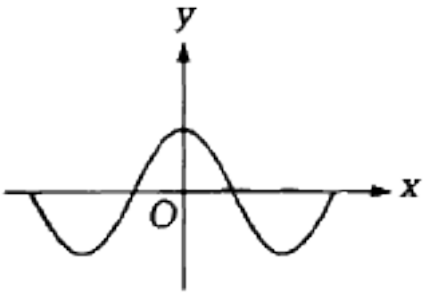
One plan for a state income tax requires those persons with income of $10,000 or less to pay no tax and those persons with income greater than $10,000 to pay a tax of 6 percent only on the part of their income that exceeds $10,000.

A person's <u>effective</u> tax rate is defined as the percent of total income that is paid in tax.

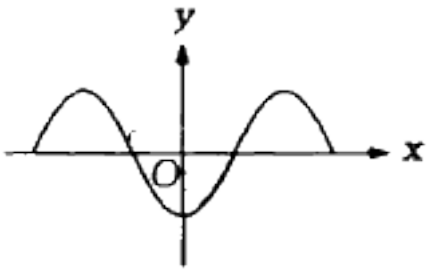
Based on this definition, could any person's effective tax rate be 5 percent? Could it be 6 percent? Explain your answer. Include examples if necessary to justify your conclusions.

1992-12-12-9

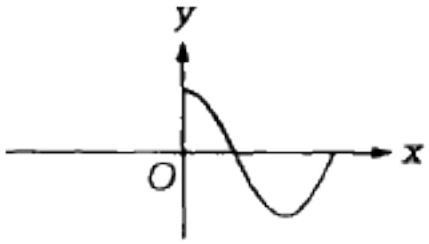
Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.



The figure above shows the graph of *y = f (x)*. Which of the following could be the graph of $y = |f(x)|$?



A.



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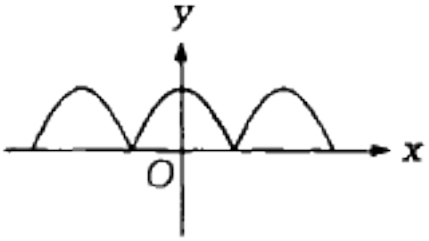
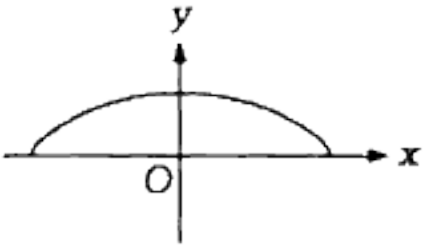
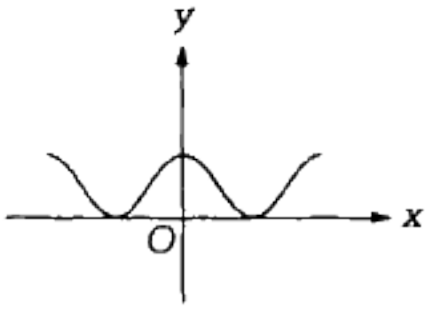


C.



D.

E. 

1992-12-15-8

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (F-BF.2.)

Sequence I: 3, 5, 9, 17, 33, . . .

Sequence I, shown above, is an increasing sequence. Each term in the sequence is greater than the previous term.

a. Make a list of numbers that consists of the positive differences between each pair of adjacent terms in Sequence I. Label the list Sequence II.

b. If this same pattern of differences continues for the terms in Sequence I, what are the next two terms after 33 in Sequence I?

6th term _____

7th term _____

c. Write an algebraic expression (rule) that can be used to determine the nth term of Sequence II, which is the difference between the $(n + 1)$st term and the nth term of Sequence I.

2005-12-3-17

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

The first term in a sequence of numbers is $\frac{1}{2}$. Each term after the first term is 1 more than twice its previous term. What is the 4th term?

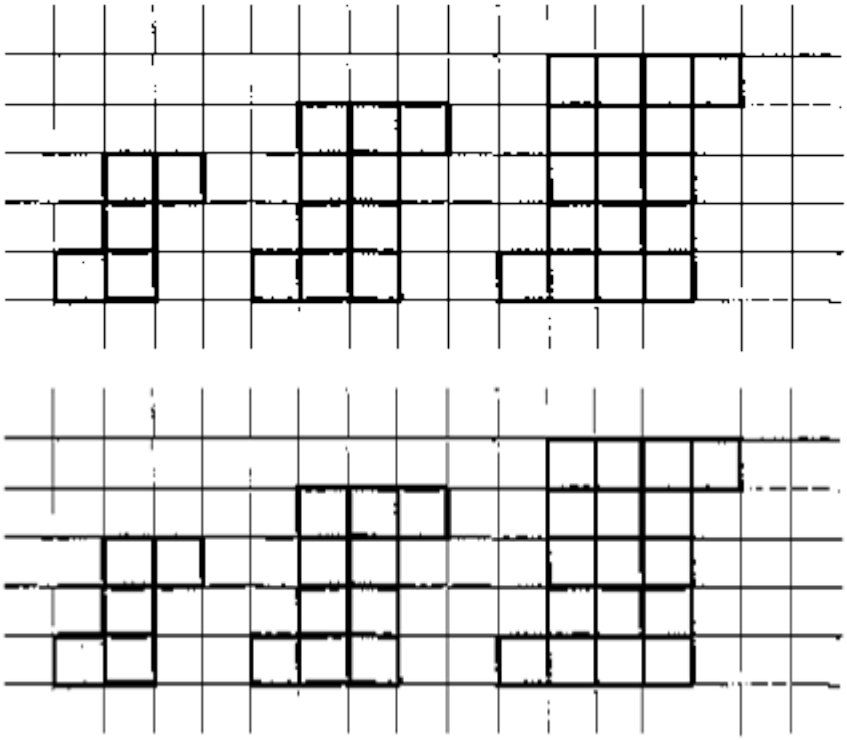
A. 2
B. 4
C. 5
D. 11
E. 23

2005-12-12-5

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

The first 3 figures in a pattern of tiles are shown below. The pattern of tiles contains 50 figures.

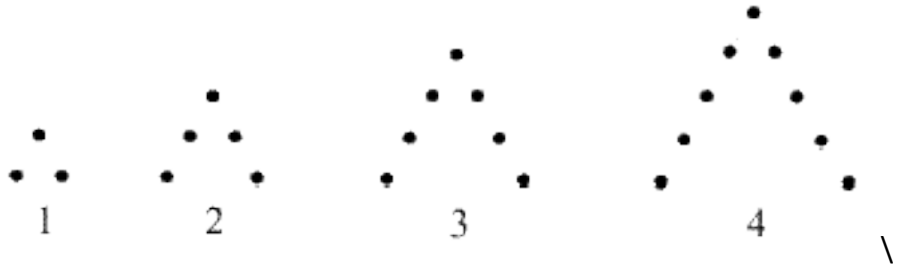


Describe the 20th figure in this pattern, including the total number of tiles it contains and how they are arranged. Then explain the reasoning that you used to determine this information. Write a description that could be used to define any figure in the pattern.

1996-12-13-9

Source: National Assessment of Educational Progress, 1996, Grade 12 Mathematics Assessment.

The following question refers to the following pattern of dot-figures.



If this pattern of dot-figures is continued, how many dots will be in the 100th figure?

A. 100
B. 101
C. 199
D. 200
E. 201

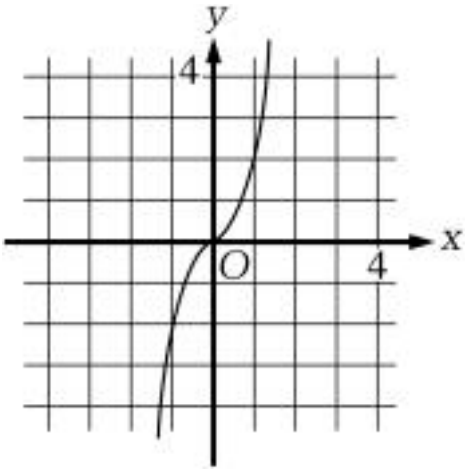
Explain how you found your answer.

1990-8-7-16&17
1990-12-7-16&17

Source: National Assessment of Educational Progress, 1990, Grade 8 and Grade 12 Mathematics Assessments.

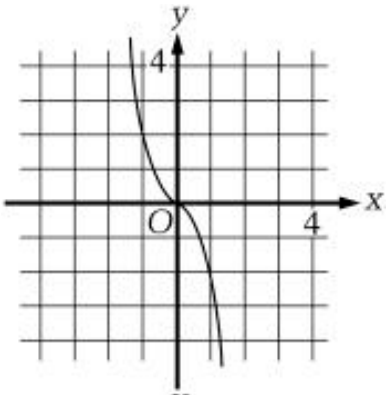
Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F-BF.3.)

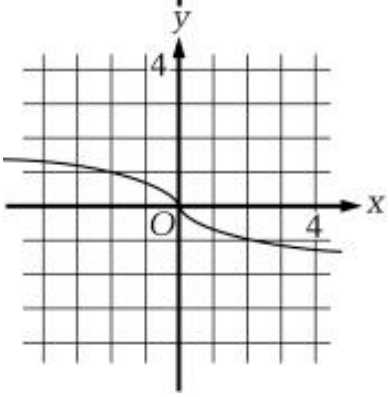


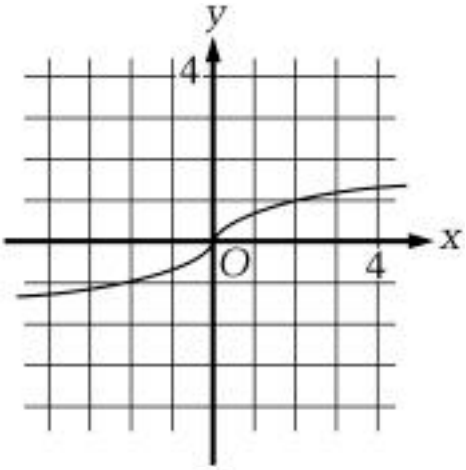
The graph of $f(x) = 2x^3$ is shown above. Which of the following is the graph of $-f(x)$?

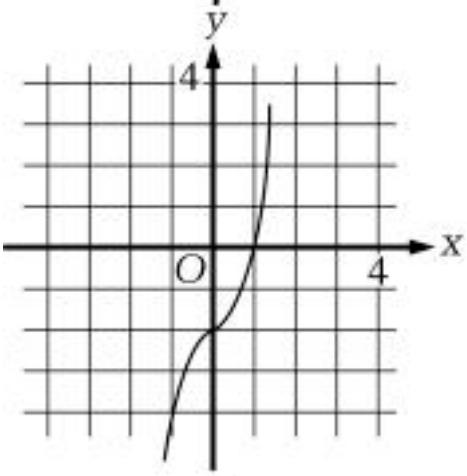
A.

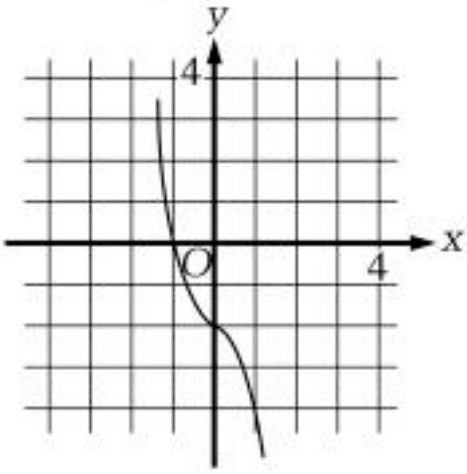


B.



C. 

D. 

E. 

2005-12-4-10

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain. (F-BF.4.)

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. (F-BF.5.)

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (F-LE.1.)

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (F-LE.2.)

It takes 28 minutes for a certain bacteria population to double. If there are 5,241,763 bacteria in this population at 1:00 p.m., which of the following is closest to the number of bacteria in millions at 2:30 p.m. on the same day?

A. 80
B. 40
C. 20
D. 15
E. 10

1992-12-5-15

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

A car costs $20,000. It decreases in value at the rate of 20 percent each year, based on the value at the beginning of that year. At the end of how many years will the value of the car first be less than half the cost?

Answer: _____ years

Justify your answer.

2005-12-12-12

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

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3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (F-LE.3.)

4. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. (F-LE.4.)

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context. (F-LE.5.)

The number of bacteria present in a laboratory sample after *t* days can be represented by $500(2^t)$. What is the initial number of bacteria present in this sample?

A. 250
B. 500
C. 750
D. 1,000
E. 2,000

2005-12-12-14

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (F-TF.1.)

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (F-TF.2.)

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number. (F-TF.3.)

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. (F-TF.4.)

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. (F-TF.5.)

Amplitude: 2

Period: $\frac{2\pi}{3}$

Which of the following trigonometric functions has the properties given above?

A. $y = \frac{2}{3}\cos(2x)$

B. $y = \frac{2}{3}\cos(3x)$

C. $y = \frac{3}{2}\cos(2x)$

D. $y = 2\cos(\frac{2}{3}x)$

E. $y = 2\cos(3x)$

2009-12-2-13

Source: National Assessment of Educational Progress, 2009, Grade 12 Mathematics Assessment.

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6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. (F-TF.6.)

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. (F-TF.7.)

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. (F-TF.8.)

cos (3 *x*) + sin (3 *x*) =

A. 0
B. 1
C. 3
D. 6
E. 9

1996-12-10-8

Source: National Assessment of Educational Progress, 1996, Grade 12 Mathematics Assessment.

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9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. (F-TF.9.)